RENORMALIZACIJA GAUGE TEORIJA

$$\mathcal{L}_B = -\frac{1}{4} F^a_{\mu\nu B} F^{\mu\nu B a} + \bar{\Psi}_B (i\not{\partial} - m_0)\Psi_B - g_B \bar{\Psi}_B \gamma^\mu T^a A^a_{\mu B} \Psi_B$$
$$- \frac{1}{2\xi_B} (\partial^\mu A_{\mu B})^2 + \partial^\mu \bar{c}^a_B \partial_\mu c^a - g_B f^{abc} \partial_\mu \bar{c}^a A^b_\mu c^c$$
$$= \frac{1}{2} A^{\mu a}_B \Box A_{\mu a B} + \dots$$

$$A^a_{\mu B} = \sqrt{Z_A}\, A^a_\mu$$
$$c^a_B = \sqrt{Z_c}\, c^a$$
$$\Psi_{iB} = \sqrt{Z_\Psi}\, \Psi_i \longrightarrow \text{renormalizovana polja}$$

$g_B, m_B, \xi_B \rightarrow$ gole konstante int, masa kvarka i ξ

$$Z_A = 1 + \delta_A$$
$$Z_\Psi = 1 + \delta_\Psi$$
$$Z_c = 1 + \delta_c$$

$$\mathcal{L}_B = -\frac{1}{4} Z_A (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)(\partial^\mu A^{\nu a} - \partial^\nu A^{\mu a}) - \frac{1}{2}\frac{Z_A}{\xi_B}(\partial^\mu A^a_\mu)^2$$
$$+ Z_c \partial^\mu \bar{c}^a \partial_\mu c^a + Z_\Psi \bar{\Psi}(i\not{\partial} - m_B)\Psi + \frac{1}{2} g Z_A^{3/2} f^{abc}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu) A^{\mu b} A^{\nu c}$$
$$- \frac{1}{4} g_B^2 Z_A^2 f^{abc} f^{ade} A^b_\mu A^c_\nu A^{\mu d} A^{\nu e} - g_B f^{abc} Z_c \sqrt{Z_A}\, \partial_\mu \bar{c}^a A^b_\mu c^c$$
$$- g_B Z_\Psi \sqrt{Z_A}\, \bar{\Psi}_i \gamma^\mu T^a_{ij} A^{\mu a} \Psi_j$$

$$\frac{Z_A}{\xi_B} = \frac{1+\delta_\xi}{\xi}$$

$$\left.\begin{aligned}
Z_\Psi m_B &= m(1+\delta_m)\\
g_B Z_A^{3/2} &= g\mu^{\varepsilon/2}(1+\delta g_3)\\
g_B^2 Z_A^2 &= g^2 \mu^\varepsilon (1+\delta g_4)\\
g_B Z_c \sqrt{Z_A} &= g\mu^{\varepsilon/2}(1+\delta g_1)\\
g_B Z_\Psi \sqrt{Z_A} &= g\mu^{\varepsilon/2}(1+\delta g_2)
\end{aligned}\right\}$$

MORA VAŽITI:

$$\frac{g_B}{g\mu^{\varepsilon/2}} = \frac{Z_3}{Z_A^{3/2}} = \frac{\sqrt{Z_4}}{Z_A} = \frac{Z_1}{Z_c\sqrt{Z_A}} = \frac{Z_2}{Z_\Psi\sqrt{Z_A}}$$

(ovo sledi iz Slavnov–Teylorovih id.

Gauge sim. je "narušena" fiksiranjem gauge. Gauge fix. lagranž. poseduje BRST simetriju

„$\mathcal{L}_R$"

$$\mathcal{L}_B = -\frac{1}{4}\left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a\right)^2 - \frac{1}{2\xi}\left(\partial_\mu A^{\mu a}\right)^2 + \partial_\mu \bar{c}^a \partial_\mu c^a + \bar{\psi}_i (i\not\partial - m)\psi_i$$
$$+ \frac{1}{2} g \mu^{\varepsilon/2} f^{abc}\left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a\right) A^{\mu b} A^{\nu c} - \frac{1}{4} g^2 \mu^{\varepsilon} f^{abc} f^{ade} A_\mu^b A_\nu^c A^{\mu d} A^{\nu e}$$
$$- g\mu^{\varepsilon/2} \partial_\mu \bar{c}^a A_\mu^b c^c f^{abc} - g\mu^{\varepsilon/2} \bar{\psi}_i \gamma^\mu T^a_{ij} A^{\mu a}\psi_j$$

$$- \frac{\delta_A}{4}\left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a\right)^2 - \frac{1}{2\xi}\delta_\xi\left(\partial_\mu A^{\mu a}\right)^2 + \delta_c \partial^\mu \bar{c}^a \partial_\mu c^c$$
$$+ \delta_\psi \bar{\psi}_i i\not\partial \psi_i - m\delta_m \bar{\psi}_i \psi_i + \frac{1}{2} g\mu^{\varepsilon/2}\delta g_3 f^{abc}\left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a\right) A^{\mu b} A^{\nu c}$$
$$- \frac{1}{4} g^2 \mu^{\varepsilon}\delta g_4 f^{abc} f^{ade} A_\mu^b A_\nu^c A^{\mu d} A^{\nu e} - g\mu^{\varepsilon/2}\delta g_1 \partial_\mu \bar{c} A_\mu^b c^c f^{abc}$$
$$- g\mu^{\varepsilon/2}\delta g_2 \bar{\psi}_i \gamma^\mu T^a_{ij} A^{\mu a}\psi_j$$

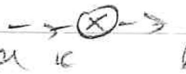
kontračlanovi

Bitno je da kvantne korekcije ne generiraju nove članove

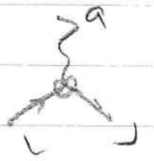
Dopunski verteksi od CT.

$\left(-i\delta_A (k^2 g_{\mu\nu} - k_\mu k_\nu) - i\frac{\delta_\xi}{\xi} k^\mu k^\nu\right)\delta^{ab}$ gauge l.

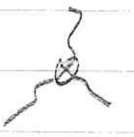
$i(\not p\,\delta_\psi - m\delta_m)\delta_{ij}$ kvark lin



$-i\delta^{ac}\delta_c k^2$

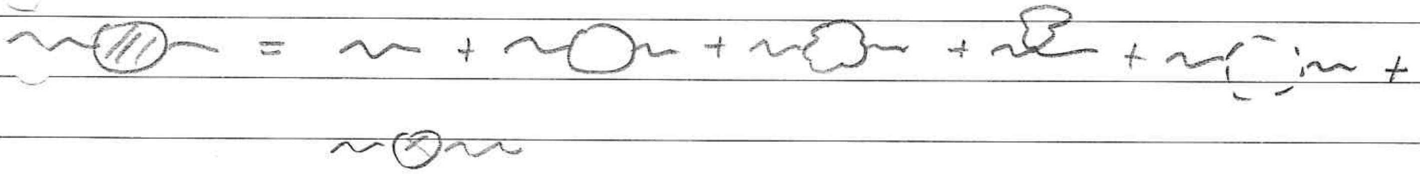


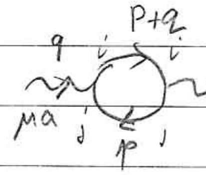
$-ig\,\delta g_2\,\mu^{\varepsilon/2} T^a_{ij}\gamma^\mu$



$-k_\mu \mu^{\varepsilon/2}\delta g_1 f^{ab}$

GLUONS



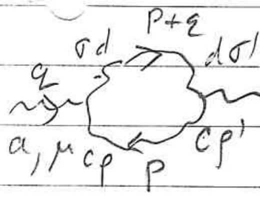


$$=(-)^2\mu^\varepsilon g^2\int\frac{d^Dp}{(2\pi)^D}\,\mathrm{Tr}\left(\frac{i}{\not p+\not q-m}\gamma^\mu\frac{i}{\not p-m}\gamma^\nu\right)T^a_{ji}T^b_{ji}\,N_f$$

$$=-g^2\mu^\varepsilon\,\mathrm{Tr}(T^aT^b)N_f\int\frac{d^Dp}{(2\pi)^D}\frac{\mathrm{Tr}\,(\not p+\not q+m)\gamma^\mu(\not p-m)\gamma^\nu}{((q+p)^2-m^2)\,(p^2-m^2)}$$

$$\boxed{=-\frac{ig^2}{16\pi^2}\,\frac{4}{3}\underbrace{N_f}_{\text{BROJ FERMINA}}\,C(r)\,\delta^{ab}\,(q^2g^{\mu\nu}-q^\mu q^\nu)\left(\frac{2}{\varepsilon}+\cdots\right)}$$

Zanima nas div dio koji je neovisan of mase fer.

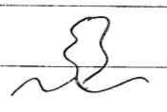


$$=\frac{g^2\mu^\varepsilon}{2}\int\frac{d^Dp}{(2\pi)^D}\frac{-1}{p^2(p+q)^2}\left[g^{\sigma\sigma'}+(\xi-1)\frac{(q+p)^\sigma(p+q)^{\sigma'}}{(q+p)^2}\right]\cdot$$

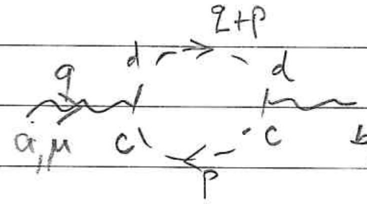
$$\left[g^{\rho\rho'}+(\xi-1)\frac{p^\rho p^{\rho'}}{p^2}\right]\left[g_{\mu\rho}(p-q)_\sigma+g_{\rho\sigma}(-2p-q)_\mu+g_{\mu\sigma}(p+2q)_\rho\right]\cdot$$

$$\cdot\left[g_{\nu\rho'}(q-p)_{\sigma'}+g_{\rho'\sigma'}(q+2p)_\nu+g_{\nu\sigma'}(-2q-p)_{\rho'}\right]f^{acd}f^{bcd}$$

$$\boxed{=\frac{ig^2}{16\pi^2\varepsilon}C_2(G)\,\delta^{ab}\left[q^2g^{\mu\nu}\left(\frac{19}{6}-\xi+1\right)-q^\mu q^\nu\left(\frac{11}{3}-\xi+1\right)\right]}$$



$$=0$$



$$=-g\mu^\varepsilon\int\frac{d^Dp}{(2\pi)^D}\frac{i}{p^2}\frac{i}{(q+p)^2}f^{dac}f^{cbd}(q+p)^\mu p^\nu$$

$$=\frac{ig^2}{16\pi^2}C_2(G)\,\delta^{ab}\left(\frac{1}{12}q^2g^{\mu\nu}+\frac{1}{6}q^\mu q^\nu\right)\left(\frac{2}{\varepsilon}+\cdots\right)$$

$$\boxed{f^{acd}f^{bcd}=C_2(G)\,\delta^{ab}}$$

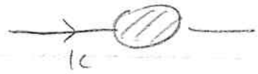
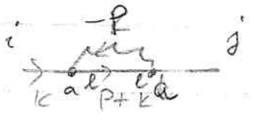
 $= -i\delta_A \left(k^2 g^{\mu\nu} - k^\mu k^\nu\right) - i\,\dfrac{\delta_\xi}{\xi}\, k^\mu k^\nu$

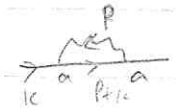
 $= \dfrac{i g^2}{16\pi^2}\, \delta^{ab} \left[-\dfrac{4}{3} N_f C(r) + \left(\dfrac{5}{3} - \dfrac{1}{2}(\xi - 1)\right) C_2(G)\right] \cdot \left(k^2 g_{\mu\nu} - k_\mu k_\nu\right)$
$- i\,\delta Z_A \left(k^2 g_{\mu\nu} - k_\mu k_\nu\right)\delta^{ab} - i\,\dfrac{\delta_\xi}{\xi}\, k^\mu k_\nu\, \delta^{ab}$

$\Rightarrow \boxed{\delta Z_A = \dfrac{g^2}{16\pi^2}\left[-\dfrac{4}{3} N_f C(r) + \left(\dfrac{5}{3} - \dfrac{1}{2}(\xi - 1)\right) C_2(G)\right] \dfrac{2}{\varepsilon}}$

$\boxed{\delta_\xi = 0}$

Quarks

 $=$  $+$ 

 $= -(-ig)^2\, T^a_{il}\, T^a_{lj}\, \mu^\varepsilon \displaystyle\int \frac{dp}{(2\pi)^D} \frac{1}{p^2 + i\varepsilon} \left(-g^{\mu\nu} + (1-\xi)\frac{p^\mu p^\nu}{p^2}\right) \gamma_\nu \frac{1}{(\not p + \not k) - m + i\varepsilon}\, \gamma_\mu$

$= \dots$

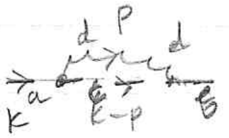
$= \dfrac{i g^2 \delta_{ij}}{16\pi^2} C_2(r) \left(\xi \not k - (3+\xi) m\right)\left(\dfrac{2}{\varepsilon} + \text{Fin. part}\right)$

$\Rightarrow \boxed{\begin{aligned} \delta Z_\psi &= -\frac{g^2}{16\pi^2} C_2(r)\, \xi \left(\frac{2}{\varepsilon} + \cancel{F.p.}\right) \\ \delta_m &= -\frac{g^2}{16\pi^2} C_2(r)(3+\xi)\left(\frac{2}{\varepsilon} + \cancel{F.p.}\right) \end{aligned}}$

MS

$(T^a T^a)_{ij} = C_2(r) \cdot \mathbb{I}$

ghost corrections

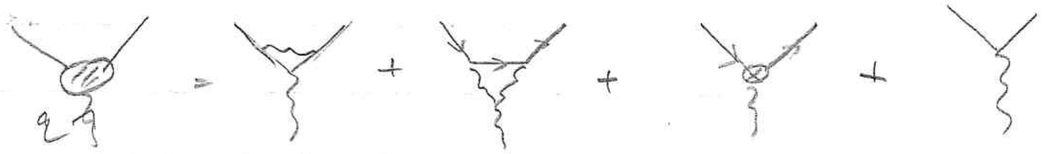
 $= g\mu^{\varepsilon} f_{dbc} f_{dcb} \int \frac{dp}{(2\pi)^D} k^{\mu} (k-p)^{\nu} \frac{-i}{(k-p)^2 + i\varepsilon} \cdot i\left(-g_{\mu\nu} + \frac{(1-\xi)p_\mu p_\nu}{p^2}\right)$

$$= \frac{ig^2}{16\pi^2}\delta^{ab} C_2(G)\frac{3-5\xi}{4} k^2 \left(\frac{2}{\varepsilon} + \text{Finite part}\right)$$

$- - \otimes - - \;= +i\,\delta^{ab} k^2 \,\delta Z_c$

$$\Rightarrow \delta Z_c = \frac{g^2}{16\pi^2} C_2(G)\frac{3-5\xi}{4}\left(\frac{2}{\varepsilon} + \text{F. p.}\right)$$

Quark - gluon coupling



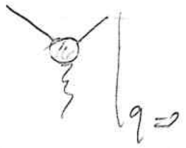
$= -\frac{ig^3}{16\pi^2}\left(C_2(r) - \frac{1}{2}C_2(G)\right)\xi\, T^a_{ji}\gamma^\mu\left(\frac{2}{\varepsilon} + \text{F.p}\right)$ ← prvi d.

$-\frac{ig^3}{16\pi^2} C_2(G)\frac{3(1+\xi)}{4} T^a_{ji}\gamma^\mu\left(\frac{2}{\varepsilon} + \text{F. p}\right)$ ← drugi d.

$- i\delta_2 g\mu^{\varepsilon/2}\gamma^\mu T^a_{ji}$ ← treći

$- ig\mu^{\varepsilon/2} T^a_{ji}$ ← četvrti

Pri izračunavanju divergencije može se uzeti [illegible] $\Rightarrow$ i $\xi = 0$

 $\Big|_{q=0} = -ig\,T^a_{ji}\,\mu^{\varepsilon/2}$

$$\Rightarrow \delta_2 = -\frac{g^2}{16\pi^2}\left(\frac{3}{4}C_2(G) + \left(\frac{1}{4}C_2(G) + C_2(r)\right)\xi\right)\left(\frac{2}{\varepsilon} + \text{F.p}\right)$$

Koristi se

$$t^b t^a t^b = t^b t^b t^a + t^b [t^a, t^b]$$
$$= C_2(r) t^a + i f^{abc} t^b t^c$$
$$= C_2(r) t^a + \frac{i}{2} f^{abc} i f^{bcd} t^d$$
$$= \left(C_2(r) - \frac{1}{2} C_2(G)\right) t^a$$

BETA FUNKCIJA

$$g_B = \frac{g\mu^{\varepsilon/2}(1+\delta_1)}{Z_\psi \sqrt{Z_A}} = g\mu^{\varepsilon/2}\left(1 + \delta_1 - \delta Z_\psi - \frac{1}{2}\delta Z_A\right)$$

$$0 = \frac{\varepsilon}{2}\mu^{\frac{\varepsilon}{2}-1} g\Big[\cancel{1} - \frac{g^2}{8\pi^2\varepsilon}\left(\frac{3}{4}C_2(G) + \frac{1}{4}\cancel{C_2(G)} + \cancel{C_2(r)}\xi\right)$$
$$+ \frac{g^2}{8\pi^2\varepsilon}\cancel{C_2(r)\xi} - \frac{1}{2}\frac{g^2}{8\pi^2\varepsilon}\left(\left(\frac{13}{6} - \cancel{\frac{3}{2}}\right)C_2(G) - \frac{4}{3}N_f C(r)\right)\Big]$$
$$+ \mu^{\frac{\varepsilon}{2}}\frac{\partial g}{\partial\mu}\left[1 - \frac{3g^2}{8\pi^2\varepsilon}\cdot\frac{3}{4}C_2(G) - \frac{1}{2}\cdot\frac{3g^2}{8\pi^2\varepsilon}\cdot\left(\frac{13}{6}C_2(G) - \frac{4}{3}N_f C(r)\right)\right]$$

$$\beta = \mu\frac{\partial g}{\partial\mu} = \frac{-\frac{1}{2}\varepsilon g\left[1 - \frac{g^2}{8\pi^2\varepsilon}\frac{3}{4}C_2(G) - \frac{1}{2}\frac{g^2}{8\pi^2\varepsilon}\left(\frac{13}{6}C_2(G) - \frac{4}{3}N_f C(r)\right)\right]}{1 - \frac{3g^2}{8\pi^2\varepsilon}\frac{3}{4}C_2(G) - \frac{1}{2}\frac{3g^2}{8\pi^2\varepsilon}\left(\frac{13}{6}C_2(G) - \frac{4}{3}N_f C(r)\right)}$$

$$= -\frac{1}{2}g\varepsilon\left[1 + \frac{2g^2}{8\pi^2\varepsilon}\frac{3}{4}C_2(G) + \frac{2}{2}\frac{g^2}{8\pi^2\varepsilon}\left(\frac{13}{6}C_2(G) - \frac{4}{3}N_f C(r)\right)\right]$$

$\frac{3}{2} + \frac{13}{6}$

$$\boxed{\beta = \frac{g^3}{16\pi^2}\left[\frac{4}{3}N_f C(r) - \frac{11}{3}C_2(G)\right]}$$

FOR $SU(N)$ grupa $C_2(\Theta) = N$; $C(1) = N$

$$\Rightarrow \beta(SU(N)) = \frac{g^3}{16\pi^2}\left(\frac{2}{3}N_f - \frac{11N}{3}\right)$$

FOR QCD $G = SU(3)$

$$\boxed{\beta = \frac{g^3}{16\pi^2}\left(\frac{2}{3}N_f - 11\right)}$$

Za $N_f < 16 \Rightarrow \beta < 0$ TEORIJA JE asimptotski slobodna

Gross, Politzer, Wilzcek

(Nobel Prize 2004)