ZADACI IZ SUPERSIMETRIJA

1. Pokazati da je sa
$$\Lambda^{\mu}{}_{\nu} = \frac{1}{2}\mathrm{Tr}\Big(\bar{\sigma}^{\mu} M \sigma_{\nu} M^{\dagger}\Big)$$
zadat homomorfizam izmedju grupa: $SL(2,C)$ and $L_{+}^{\uparrow}$, tj. pokazati da važi
$$\Lambda^{\mu}{}_{\nu}(M)\Lambda^{\nu}{}_{\rho}(N) = \Lambda^{\mu}{}_{\rho}(MN) \ .$$
Odrediti kernel ovog homomorfizma. Množeći
$$\Lambda^{\mu}{}_{\nu}\sigma_{\mu} = M\sigma_{\nu}M^{\dagger}$$
sa $\bar{\sigma}^{\nu}$ pokazati da je
$$M = \frac{\Lambda^{\mu}{}_{\nu}\sigma_{\mu}\bar{\sigma}^{\nu}}{2\mathrm{tr}(M^{\dagger})}$$
Pokazati takodje da je
$$\mathrm{tr}(M^{\dagger}) = \pm\frac{1}{2}\sqrt{\det(\Lambda^{\mu}{}_{\nu}\sigma_{\mu}\bar{\sigma}^{\nu})} \ ,$$
odnosno
$$M(\Lambda) = \pm\frac{\Lambda^{\mu}{}_{\nu}\sigma_{\mu}\bar{\sigma}^{\nu}}{\sqrt{\det(\Lambda^{\mu}{}_{\nu}\sigma_{\mu}\bar{\sigma}^{\nu})}} \ .$$

2. Pokazati da su $\epsilon_{\alpha\beta}$ i $\epsilon_{\dot{\alpha}\dot{\beta}}$ invarijantni tenzori pri Lorencovim transformacijama.

3. Pokazati da je $\psi\sigma^{\mu}\bar{\chi}$ Lorencov vektor. Razmotriti dva slučaja. U prvom uzeti da su ψ i χ Grasmanove funkcije a u drugom da su to operatori polja.

4. Pokazati da se ψ_{α} transformiše po $\left(\frac{1}{2},0\right)$, a $\bar{\psi}^{\dot{\alpha}}$ po $\left(0,\frac{1}{2}\right)$ ireducibilnim reprezentacijama Lorencove grupe.

5. Pokazati

   (a) $\varphi\sigma^{\mu}\bar{\chi} = -\bar{\chi}\bar{\sigma}^{\mu}\varphi$

   (b) $\varphi\sigma^{\mu\nu}\chi = -\chi\sigma^{\mu\nu}\varphi$

   (c) $\bar{\varphi}\bar{\sigma}^{\mu\nu}\bar{\chi} = -\bar{\chi}\bar{\sigma}^{\mu\nu}\bar{\varphi}$

   (d) $(\varphi\sigma^{\mu}\bar{\chi})^{\dagger} = \chi\sigma^{\mu}\bar{\varphi}$

(e) $\varphi\sigma^\mu\bar{\sigma}^\nu\chi = \chi\sigma^\nu\bar{\sigma}^\mu\varphi$

(f) $(\chi\sigma^{\mu\nu}\psi)^\dagger = \bar{\psi}\bar{\sigma}^{\mu\nu}\bar{\chi}$

(g) $(\bar{\varphi}\bar{\sigma}^{\mu\nu}\bar{\chi})^\dagger = \chi\sigma^{\mu\nu}\varphi$.

6. Pokazati $(\sigma^{\mu\nu})_\alpha^{\ \alpha} = 0$ i $(\bar{\sigma}^{\mu\nu})^{\dot{\alpha}}_{\ \dot{\alpha}} = 0$.

7. Pokazati $(\sigma^{\mu\nu})_\alpha^{\ \beta}\epsilon_{\beta\gamma} = (\sigma^{\mu\nu})_\gamma^{\ \beta}\epsilon_{\beta\alpha}$.

8. Polazeći od

$$\gamma^\mu\gamma^\nu\gamma^\rho = (g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\rho\nu})\gamma^\sigma + i\epsilon^{\sigma\mu\nu\rho}\gamma_5\gamma_\sigma \tag{0.1}$$

pokazati

$$\begin{aligned} \sigma^\mu\bar{\sigma}^\nu\sigma^\rho &= g^{\mu\nu}\sigma^\rho - g^{\mu\rho}\sigma^\nu + g^{\nu\rho}\sigma^\mu - i\epsilon^{\sigma\mu\nu\rho}\sigma_\sigma \\ \bar{\sigma}^\mu\sigma^\nu\bar{\sigma}^\rho &= g^{\mu\nu}\bar{\sigma}^\rho - g^{\mu\rho}\bar{\sigma}^\nu + g^{\nu\rho}\bar{\sigma}^\mu + i\epsilon^{\sigma\mu\nu\rho}\bar{\sigma}_\sigma \ . \end{aligned} \tag{0.2}$$

Pokazati identitete sa tragovima:

$$\mathrm{tr}(\sigma^{\mu\nu}\sigma^{\rho\sigma}) = \frac{1}{2}(g^{\mu\rho}g^{\nu\sigma} - g^{\nu\rho}g^{\mu\sigma} - i\epsilon^{\mu\nu\rho\sigma}) \tag{0.3}$$

$$\mathrm{tr}(\bar{\sigma}^{\mu\nu}\bar{\sigma}^{\rho\sigma}) = \frac{1}{2}(g^{\mu\rho}g^{\nu\sigma} - g^{\nu\rho}g^{\mu\sigma} + i\epsilon^{\mu\nu\rho\sigma}) \ . \tag{0.4}$$

9. Pokazati:

(a) $\theta^\alpha\theta^\beta = -\frac{1}{2}\epsilon^{\alpha\beta}\theta\theta$

(b) $\theta_\alpha\theta_\beta = \frac{1}{2}\epsilon_{\alpha\beta}\theta\theta$

(c) $\bar{\theta}^{\dot{\alpha}}\bar{\theta}^{\dot{\beta}} = \frac{1}{2}\epsilon^{\dot{\alpha}\dot{\beta}}\bar{\theta}\bar{\theta}$

(d) $\bar{\theta}_{\dot{\alpha}}\bar{\theta}_{\dot{\beta}} = -\frac{1}{2}\epsilon_{\dot{\alpha}\dot{\beta}}\bar{\theta}\bar{\theta}$.

10. Proveriti da je $\sigma^{\mu\nu}$ samodualni a $\bar{\sigma}^{\mu\nu}$ antisamodualni tenzor, tj.

$$\epsilon_{\mu\nu\rho\sigma}\sigma^{\rho\sigma} = 2i\sigma_{\mu\nu} \ ,$$

$$\epsilon_{\mu\nu\rho\sigma}\bar{\sigma}^{\rho\sigma} = -2i\bar{\sigma}_{\mu\nu} \ .$$

11. Dokazati (Fierz-ovi identiteti)

(a) $(\theta\sigma^\mu\bar{\theta})(\theta\sigma^\nu\bar{\theta}) = \frac{1}{2}\eta^{\mu\nu}(\theta\theta)(\bar{\theta}\bar{\theta})$

(b) $(\bar{\theta}\bar{\lambda})(\chi\sigma^\mu\bar{\theta}) = -\frac{1}{2}(\bar{\theta}\bar{\theta})(\chi\sigma^\mu\bar{\lambda})$

(c) $(\theta\sigma^\nu\bar{\psi})(\theta\sigma^\mu\bar{\lambda}) = \frac{1}{2}(\theta\theta)(\bar{\psi}\bar{\sigma}^\nu\sigma^\mu\bar{\lambda})$

12. Izraziti $\bar{\Psi}\gamma^\mu\gamma_5\Phi$ preko Vejlovih spinora ako su Ψ i Φ Dirakovi spinori. Uraditi isto ako se radi o Majorana spinorima.

13. Napisati i proveriti važenje super-Jakobijevog identiteta za sledeća tri operatora $B, F_1, F_2;\ \ F_1, F_2, F_3$.

14. Neka je $X_{\mu\nu}$ tenzor drugog reda. Prelaskom na spinorske indekse dobijamo

$$X^{\mu\nu} \to X^{\alpha\beta\dot{\alpha}\dot{\beta}}\sigma^\mu_{\alpha\dot{\alpha}}\sigma^\nu_{\beta\dot{\beta}} \ .$$

Pokazati da su ireducibilne komponente dobijenog tenzora

$$X_{\alpha\beta\dot{\alpha}\dot{\beta}} = \epsilon_{\alpha\beta}X_{(\dot{\alpha}\dot{\beta})} + \epsilon_{\dot{\alpha}\dot{\beta}}X_{(\alpha\beta)} + \epsilon_{\alpha\beta}\epsilon_{\dot{\alpha}\dot{\beta}}X + X_{(\alpha\beta)(\dot{\alpha}\dot{\beta})}$$

gde je

$$\begin{aligned} X &= \frac{1}{4}\epsilon_{\alpha\beta}\epsilon_{\dot{\alpha}\dot{\beta}}X^{\alpha\beta\dot{\alpha}\dot{\beta}} \\ X_{(\alpha\beta)} &= -\frac{1}{2}\epsilon^{(\dot{\alpha}\dot{\beta})}X_{(\alpha\beta)\dot{\alpha}\dot{\beta}} \\ X_{(\dot{\alpha}\dot{\beta})} &= -\frac{1}{2}\epsilon^{\alpha\beta}X_{\alpha\beta(\dot{\alpha}\dot{\beta})} \ . \end{aligned} \qquad (0.5)$$

Primenom gornjih formula naći ireducibilne spinorske komponente tenzora $g_{\mu\nu}$ i tenzora jačine polja $F_{\mu\nu}$

15. Pokazati da komutator dve supersimetrične transformacije daje translaciju

$$[\delta_\xi, \delta_\eta]\psi_\alpha = i\epsilon^\mu\partial_\mu\psi_\alpha \ ,$$

gde parametar ϵ^μ treba odrediti.

Zakoni transformacija su

$$\begin{aligned} \delta A &= \sqrt{2}\xi\psi, \\ \delta\psi_\alpha &= \sqrt{2}F\xi_\alpha - i\sqrt{2}\partial_m A\sigma^m_{\alpha\dot{\alpha}}\bar{\xi}^{\dot{\alpha}}, \\ \delta_\xi F &= i\sqrt{2}\partial_m\psi\sigma^m\bar{\xi}. \end{aligned} \qquad (0.6)$$

16. Pokazati da je varijacija slobodnog Wess-Zumino lagranžijana

$$\mathcal{L} = -A^*\Box A + i\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi + F^*F \qquad (0.7)$$

pri supersimetričnim transformacijama totalna divergencija.

17. Naći zakon transformacije člana $2\varphi F - \psi\psi$ pri supersimetričnim transformacijama.

18. Izračunati komutator supernaboja $\bar{Q}_{\dot{\alpha}}$ sa vektorom Pauli-Lubanskog $W_\mu = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}M^{\nu\rho}P^\sigma$.

19. Pokazati da kod bezmasenog supersimetričnog multipleta generatori $Q_1, Q_2, \bar{Q}_{\dot{1}}, \bar{Q}_{\dot{2}}$ menjaju helicitet za $-\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}$ respektivno.

20. Pokazati da je $[\mathbf{J}, \bar{Q}^{\dot{\alpha}}] = -\frac{1}{2}\boldsymbol{\sigma}^{\dot{\alpha}}{}_{\dot{\beta}}\bar{Q}^{\dot{\beta}}$.

21. Ako je maksimalna vrednost heliciteta bezmasenog $N = 8$ multipleta $\lambda_{\max} = 2$ odrediti koja sve stanja sadrži ovaj multiplet.

22. Generatore unutrašnje simetrije obeležili smo sa B^l. Pokazati da iz $[B^l, Q^A_\alpha] = -(b^l)^A{}_B Q^B_\alpha$ sledi $[B^l, \bar{Q}^A_{\dot{\alpha}}] = (b^{l\star})^A{}_B \bar{Q}^B_{\dot{\alpha}}$, gde su b^l matrice. Polazeći od super-Jakobijevog identiteta za operatore (B^r, B^l, Q^C_α) pokazati da matrice b^l zadovoljavaju Lijevu algebru $[b^r, b^l] = if^{rlm}b^m$. Takodje, polazeći od super-Jakobijevog identiteta za $(Q^A_\alpha, \bar{Q}^B_{\dot{\alpha}}, B^l)$ sledi da su ove matrice hermitske.

23. U ovom zadatku analiziraćemo proširenu supersimetriju. Pokazaćemo da centralno naelektrisanje komutira sa svim generatorima superalgebre, tako da čine abelovu invarijantnu podalgebru.

    (a) Polazeći od super-Jakobijevog identiteta za operatore $(B^l, Q^A_\alpha, Q^B_\beta)$ pokazati da važi
    $$[B^l, Z^{AB}] = -(b^l)^B{}_C Z^{AC} + (b^l)^A{}_C Z^{AB} \ .$$
    Pošto je $Z^{AB} = \lambda^{ABl}B^l$ onda iz prethodnog rezultata lako sledi
    $$[Z^{CD}, Z^{AB}] = \lambda^{CDl}(-(b^l)^B{}_C Z^{AC} + (b^l)^A{}_C Z^{AB}) \ .$$

    (b) Polazeći od super-Jakobijevog identiteta za operatore $(Q^A_\alpha, Q^B_\beta, \bar{Q}^C_{\dot{\alpha}})$ pokazati da je $[\bar{Q}^C_{\dot{\alpha}}, Z^{AB}] = 0$.

    (c) Iz rezultata dobijenog u delu (b) zaključiti da je $\lambda^{ABr}(b^r)^C{}_D = 0$ što znači da je $[Z^{CD}, Z^{AB}] = 0$.

24. Lagranžijan bezmasenog Wess-Zumino modela je
$$\mathcal{L} = \partial_\mu A^\dagger \partial^\mu A + \frac{i}{2}\left(\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi + \psi\bar{\sigma}^\mu\partial_\mu\bar{\psi}\right) + F^\dagger F \ .$$

Pokazati da je Neterina struja za supersimetrične transformacije

$$\begin{aligned} j^\mu_\alpha &= \sqrt{2}(\sigma^\nu\bar{\sigma}^\mu\psi)_\alpha\partial_\nu A^\dagger \\ \bar{j}^\mu_{\dot{\alpha}} &= \sqrt{2}(\bar{\psi}\bar{\sigma}^\mu\sigma^\nu)_{\dot{\alpha}}\partial_\nu A \ . \end{aligned} \tag{0.8}$$

Odrediti supersimetrične generatore $Q_\alpha, \bar{Q}_{\dot{\alpha}}$ i pokazati da je

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2\sigma^\mu_{\alpha\dot{\alpha}}P_\mu \ . \tag{0.9}$$

25. Naći $-(a^\mu\partial_\mu + \xi^\alpha Q_\alpha + \bar{\xi}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}})x^\nu$ kao i $[\delta_\eta, \delta_\xi]x^\nu$.

26. Pod dejstvom simetrije polja ϕ_n transformišu se prema

$$\phi'_n = U_\theta^{-1}\phi_n U_\theta = e^{-i\theta^a Q^a}\phi_n e^{i\theta^a Q^a} = \left(e^{i\theta^a T^a}\right)_{nj}\phi_j \ , \tag{0.10}$$

gde su Q^a operatori simetrije, a T^a su matrice koje zadovoljavaju istu Lijevu algebru kao i naboji:

$$[Q^a, Q^b] = if^{abc}Q^c, \quad [T^a, T^b] = if^{abc}T^c \ . \tag{0.11}$$

Iz zakona transformacije je jasno da je infinitezimalna transformacija polja data sa

$$\delta_\theta\phi_n = -[i\theta^a Q^a, \phi_n] = i\theta^a T^a_{nj}\phi_j \ . \tag{0.12}$$

(a) Pokazati da infinitezimalno važi

$$U_\theta U_\epsilon = 1 + i\theta^a Q^a + i\epsilon^a Q^a - (\theta^a Q^a)(\epsilon^b Q^b) \tag{0.13}$$

(b) Primenimo dve sukcesivne transformacije, prvo sa parametrom θ a zatim sa ϵ. Pokazati da je

$$U_\epsilon^{-1}U_\theta^{-1}\phi_n U_\theta U_\epsilon \approx \phi_n + \delta_\theta\phi_n + \delta_\epsilon\phi_n + \delta_\epsilon\delta_\theta\phi_n \ , \tag{0.14}$$

gde je

$$\delta_\epsilon\delta_\theta\phi_n = [-i\epsilon^a Q^a, [-i\theta^b Q^b, \phi_n]] \ . \tag{0.15}$$

(c) Pokazati da važi

$$\delta_\epsilon\delta_\theta\phi_n = i\theta^a T_{nj}(i\epsilon^b T^b_{jm}\phi_m) \ . \tag{0.16}$$

(d) Pokazati da je

$$[\delta_\epsilon, \delta_\theta]\phi_n = \delta_{-i[\epsilon,\theta]}\phi_n \ . \tag{0.17}$$

27. Ako je element supergrupe (do na Lorencove transformacije)

$$G(x,\theta,\bar\theta) = e^{i(x^\mu P_\mu + \theta Q + \bar\theta\bar Q)}$$

naći proizvod $G(x,\theta,\bar\theta)G(a,\xi,\bar\xi)$. Ovo množenje koji reprezentuje desno dejstvo grupe na superprostoru, indukuje transformaciju superkoordinata. Naći ovu transformaciju i pokazati da važi

$$\delta z^M = (a^\mu\partial_\mu + \xi^\alpha D_\alpha + \bar\xi_{\dot\alpha}\bar D^{\dot\alpha}) z^M \ .$$

28. Superkovarijantni izvodi su dati sa

$$D_\alpha = \partial_\alpha - i\sigma^\mu_{\alpha\dot\beta}\bar\theta^{\dot\beta}\partial_\mu$$

$$\bar D_{\dot\alpha} = -\bar\partial_{\dot\alpha} + i\theta^\beta\bar\sigma^\mu_{\beta\dot\alpha}\partial_\mu \ .$$

Naći $\{D_\alpha, \bar D_{\dot\beta}\}$, $\{D_\alpha, D_\beta\}$, $\{\bar D_{\dot\alpha}, \bar D_{\dot\beta}\}$, $\{D_\alpha, \bar Q_{\dot\beta}\}$.

29. Kiralno superpolje je

$$\Phi(y,\theta) = A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y) \ ,$$

gde je $y^\mu = x^\mu - i\theta\sigma^\mu\bar\theta$. Izraziti ga preko koordinata $x, \theta, \bar\theta$ ekspanzijom gornjeg izraza.

30. Polazeći od zakona transformacije kiralnog superpolja

$$\delta\Phi = (\xi Q + \bar\xi\bar Q)\Phi \ ,$$

gde su diferencijalni operatori Q i $\bar Q$ dati sa

$$Q_\alpha = \partial_\alpha + i\sigma^\mu_{\alpha\dot\beta}\bar\theta^{\dot\beta}\partial_\mu \ ,$$

$$\bar Q_{\dot\alpha} = -\bar\partial_{\dot\alpha} - i\theta^\beta\bar\sigma^\mu_{\beta\dot\alpha}\partial_\mu$$

izvesti zakone transformacije komponenti kiralnog multipleta.

31. Pokazati $\bar D_{\dot\alpha} y^\mu = 0$ i $D_\alpha \bar y^\mu = 0$.

32. Pokazati da superkovarijantni izvodi preko koordinata $(y^\mu, \theta, \bar\theta)$ imaju oblik

$$\begin{aligned} D_\alpha &= \frac{\partial}{\partial\theta^\alpha} - 2i(\sigma^\mu\bar\theta)_\alpha\frac{\partial}{\partial y^\mu} \\ \bar D_{\dot\alpha} &= -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} \ . \end{aligned} \tag{0.18}$$

33. Pokazati $\bar{D}^2 D^2 \Phi = -16\Box\Phi$.

34. Kinetički supermultiplet $T\Phi$ definisan je sa

$$T\Phi = \frac{1}{4}\bar{D}^2\bar{\Phi} \ .$$

To je očigledno kiralno superpolje. Naći njegove komponete $(\mathcal{A}, \Psi, \mathcal{F})$.

35. Pokazati $D^\alpha W_\alpha = \bar{D}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}}$.

36. Izračunati $\Phi^{*\iota}(V_{WZ})_i{}^j\Phi_j$ u slučaju neabelove gauge teorije. Odrediti takodje

$$\int d^4x (\Phi^{*\iota}(V_{WZ})_i{}^j\Phi_j)\Big|_{\theta\theta\bar{\theta}\bar{\theta}} \ .$$

37. Pokazati da je veličina $W_\alpha = -\frac{1}{4}\bar{D}^2 D_\alpha V$ gauge invarijantna.

38. Pokazati da postoji supergauge transformacija vektorskog potencijala koja očuvava Wess-Zumino gauge.

39. Za superpotencijal $W = m\phi_2\phi_3 + \frac{y}{2}\phi_1\phi_3^2$ izračunati potencijal V. Pokazati da je vakuum $\langle A_1\rangle = v$, $\langle A_2\rangle = 0$, $\langle A_3\rangle = 0$. Razviti oko vakuuma i naci bozonsku masenu matricu M^2, kao i fermionsku masenu matricu.

40. Neka su $\phi_\pm = (A_\pm, \psi_\pm, F_\pm)$, $\phi = (A, \psi, F)$ tri kiralna superpolja. Dejstvo koje poseduje globalnu $U(1)$ simetriju je dato sa

$$S = \int d^4x (\bar{\phi}_+\phi_+ + \bar{\phi}_-\phi_- + \bar{\phi}\phi)\big|_{\theta\theta\bar{\theta}\bar{\theta}} + \int d^4x (W(\phi, \phi_+, \phi_-) + c.c.) \ ,$$

gde je superpotencijal dat sa

$$W = (\frac{1}{2}m\phi^2 + \mu\phi_+\phi_- + \lambda\phi + g\phi\phi_+\phi_-)\Big|_{\theta\theta} \ .$$

Dejstvo je invarijantno na globalne $U(1)$ transformacije: $\delta\phi_\pm = \pm i\alpha\phi_\pm$, $\delta\phi = 0$.

(a) Naći jednačine za pomoćna polja i odrediti potencijal V.

(b) Pokazati da postoje dva rešenja koja ne narušavaju SUSY: Prvo je $A_+ = A_- = 0, A = -\frac{\lambda}{m}$. Drugo je $A_+A_- \neq 0$. Da li ova dva osnovna stanja narušavaju $U(1)$ simetriju?

(c) Napisati dejstvo dobijeno lokalizacijom $U(1)$ simetrije uvodeći vektorsko superpolje $V = (v_\mu, \lambda, D)$. Uključiti i član $2\kappa V$. Raspisati dejstvo u komponentama.

(d) Odrediti jednačine za pomoć na polja za dejstvo dobijeno u delu (c).

(e) Naći potencijal i jednačine za njegove stacionarne tačke.

(f) Za minimum $A_+ = A_- = 0, A = -\frac{\lambda}{m}$ ispitati narušenje SUSY i lokalne $U(1)$ simetrije. Pokazati da je λ_a Goldstonov fermion. Odrediti mase polja u teoriji i proveriti masenu formulu.

(g) Ispitati narušenje simetrije u drugoj stacionarnoj tački $A = -\frac{\mu}{g}$, $A_+A_- = -\frac{1}{g}\left(\lambda - \frac{m\mu}{g}\right)$.

41. Na času je pokazan zakon transformacije vektorskog polja u Ves-Zumino gauge pri transformacijama koje očuvavaju WZ gauge

$$\delta V_{WZ} = i(\Lambda - \Lambda^\dagger) + \frac{i}{2}[V_{WZ}, \Lambda + \Lambda^\dagger] \ . \tag{0.19}$$

Polazeći od ovog izraza na času je nadjeno δv_μ^a; nadjite $\delta\lambda$ i δD.

42. Kiralno polje se pri R transformacijama transformiše prema

$$\delta_R \Phi = i\varphi \hat{R} \Phi \ ,$$

gde je $\hat{R} = (R_\Phi - \theta^\alpha \partial_\alpha - \bar{\theta}_{\dot{\alpha}} \bar{\partial}^{\dot{\alpha}})\Phi$. Izračunati komutatore $[iQ_\alpha, \hat{R}]$ i $[i\bar{Q}_{\dot{\alpha}}, \hat{R}]$.

43. Bezmaseni Wess Zumino model je dat sa

$$S = \int d^4x(-A^*\Box A + i\bar{\psi}\bar{\sigma}^\mu \partial_\mu \psi + F^*F + g(A^2F - A\psi\psi + A^{*2}F^* - A^*\bar{\psi}\bar{\psi}))$$

(a) Pokazati da je on invarijantan na

$$\begin{aligned} \delta_R A &= \frac{2i}{3}\varphi A \\ \delta_R \psi &= -\frac{i}{3}\varphi A \\ \delta_R F &= -\frac{4i}{3}\varphi A \ . \end{aligned}$$

(b) Odrediti Neterinu struju i pokazati da je njena divergencija nula.

(c) Izračunati komutator $[R, Q_\alpha]$, gde je R očuvano naelektrisanje pri R transformacijama.

44. Dokazati sledeće identitete:

(a) $\bar{D}^2 D^2 \bar{D}^2 = -16\Box \bar{D}^2$

(b) $D^2 \bar{D}^2 D^2 = -16\Box D^2$

(c) $D\sigma^\mu \bar{D} + \bar{D}\bar{\sigma}^\mu \bar{D} = 4i\partial^\mu$

(d) $[D_\alpha, \bar{D}^2] = 4i\sigma^\mu_{\alpha\dot{\alpha}} \bar{D}^{\dot{\alpha}} \partial_\mu$

(e) $[\bar{D}_{\dot{\alpha}}, D^2] = -4iD^\beta \sigma^\mu_{\beta\dot{\alpha}} \partial_\mu$

(f) $[D^2, \bar{D}^2] = 8iD\sigma^\mu \bar{D}\partial_\mu + 16\Box$

(g) $D^2 \bar{D}_{\dot{\alpha}} D^2 = \bar{D}^2 D_\alpha \bar{D}^2 = 0$

45. Proveriti

$$\begin{aligned} D_1^2(\theta_1 - \theta_2)^2 &= -4e^{i(\theta_1-\theta_2)\sigma^\mu\bar{\theta}_1\partial_\mu} \\ \bar{D}_1^2(\bar{\theta}_1 - \bar{\theta}_2)^2 &= -4e^{-i\theta_1\sigma^\mu(\bar{\theta}_1-\bar{\theta}_2)\partial_\mu} \\ \bar{D}_1^2 D_1^2(\theta_1 - \theta_2)^2(\bar{\theta}_1 - \bar{\theta}_2)^2 &= 16e^{-i(\theta_1\sigma^\nu\bar{\theta}_1+\theta_2\sigma^\nu\bar{\theta}_2-2\theta_1\sigma^\nu\bar{\theta}_2)\partial^1_\nu} \\ D_1^2 \bar{D}_1^2(\theta_1 - \theta_2)^2(\bar{\theta}_1 - \bar{\theta}_2)^2 &= 16e^{i(\theta_1\sigma^\nu\bar{\theta}_1+\theta_2\sigma^\nu\bar{\theta}_2-2\theta_1\sigma^\nu\bar{\theta}_2)\partial^1_\nu} . \end{aligned} \tag{0.20}$$

46. Matrica $\mathcal{M}$ data je sa

$$\mathcal{M} = \begin{pmatrix} -\frac{m}{4}\bar{D}^2 & (-P_2 - \xi(P_1 + P_T))\Box \\ (-P_1 - \xi(P_2 + P_T))\Box & -\frac{m}{4}D^2 \end{pmatrix} \delta^{(8)}(z - z') . \tag{0.21}$$

Proveriti da je njena inverzna matrica data sa

$$\mathcal{M}^{-1} = \begin{pmatrix} -\frac{m}{\Box+m^2}\frac{D^2}{4\Box} & -\frac{1}{\xi}\frac{P_2+P_T}{\Box} - \frac{P_1}{\Box+m^2} \\ -\frac{1}{\xi}\frac{P_1+P_T}{\Box} - \frac{P_2}{\Box+m^2} & -\frac{m}{\Box+m^2}\frac{\bar{D}^2}{4\Box} \end{pmatrix} \delta^{(8)}(z-z') . \tag{0.22}$$

47. Pokazati

$$\int d^8z(\bar{D}_{\dot{\alpha}}\bar{S})\Big(\frac{3}{16}\bar{D}^{\dot{\alpha}}D^\alpha + \frac{1}{4}D^\alpha\bar{D}^{\dot{\alpha}}\Big)D_\alpha S = \int d^8z\bar{S}(P_2 + P_T)\Box S . \tag{0.23}$$

48. Na predavanjima je izračunati sledeći superpropagatori

$$\begin{aligned}\langle 0|TS(z_1)S(z_2)|0\rangle &= -i\frac{m}{\Box_1+m^2}\frac{D_1^2}{4\Box_1}\delta^{(8)}(z_1-z_2)\\ \langle 0|T\bar{S}(z_1)\bar{S}(z_2)|0\rangle &= -i\frac{m}{\Box_1+m^2}\frac{\bar{D}_1^2}{4\Box_1}\delta^{(8)}(z_1-z_2)\\ \langle 0|TS(z_1)\bar{S}(z_2)|0\rangle &= i\Big(-\frac{1}{\xi}\frac{P_2+P_T}{\Box_1}-\frac{P_1}{\Box_1+m^2}\Big)\delta^{(8)}(z_1-z_2)\\ \langle 0|T\bar{S}(z_1)S(z_2)|0\rangle &= i\Big(-\frac{1}{\xi}\frac{P_1+P_T}{\Box_1}-\frac{P_2}{\Box_1+m^2}\Big)\delta^{(8)}(z_1-z_2)\ .\end{aligned}$$

Primenom

$$\Phi=-\frac{1}{4}\bar{D}^2S,\quad \bar{\Phi}=-\frac{1}{4}D^2\bar{S} \tag{0.24}$$

pokazati sledeće propagatore

$$\begin{aligned}\langle 0|T\Phi(z_1)\Phi(z_2)|0\rangle &= \frac{im}{4}\frac{\bar{D}_1^2}{\Box_1+m^2}\delta^{(8)}(z_1-z_2)\\ \langle 0|T\bar{\Phi}(z_1)\bar{\Phi}(z_2)|0\rangle &= \frac{im}{4}\frac{D_1^2}{\Box_1+m^2}\delta^{(8)}(z_1-z_2)\\ \langle 0|T\Phi(z_1)\bar{\Phi}(z_2)|0\rangle &= -i\frac{\bar{D}_1^2D_1^2}{\Box_1+m^2}\delta^{(8)}(z_1-z_2)\\ \langle 0|T\bar{\Phi}(z_1)\Phi(z_2)|0\rangle &= -i\frac{D_1^2\bar{D}_1^2}{\Box_1+m^2}\delta^{(8)}(z_1-z_2)\ .\end{aligned}$$

49. Polazeći od izraza sa superpropagatore odrediti

$$\langle 0|A(x_1)A^\star(x_2)|0\rangle,\ \langle 0|\psi_\alpha(x_1)\bar{\psi}_{\dot{\beta}}(x_2)|0\rangle,\ \langle 0|A(x_1)F(x_2)|0\rangle.$$

50. Pokazati da je $\langle\Omega|\Phi(z)|\Omega\rangle=0$ u WZ modelu, tj. da tadepole dijagram isčezava.

51. Razmotriti dijagram sa jednim $\frac{\lambda}{3!}\Phi^3$ verteksa i dva $\frac{\lambda}{3!}\bar{\Phi}^3$. Izračunati verteksni dijagram koji ima jedno kiralno i dva antikiralna superpolja.

52. Dokazati identitet

$$\gamma^{\mu_1\mu_2}\gamma_{\nu_1\dots\nu_D}=D(D-1)\delta^{\mu_2\mu_1}_{[\nu_1\nu_2}\gamma_{\nu_3\dots\nu_D]}$$

53. Pokazati da u $D=8$ ne postoje Majorana spinori, a postoje pseudo-Majorana spinori. Da li postoje ovi spinori u $D=9$?

54. Na osnovu

$$[D_\mu, D_\nu]\psi = -\frac{i}{2} R_{\mu\nu}{}^{ab} \Sigma_{ab} \psi$$

naći tenzor krivine, $R_{\mu\nu}{}^{ab}$. Izračunati

$$[\nabla_\mu, \nabla_\nu]\psi_\mu \ .$$

55. Spinska koneksija je data sa

$$\omega_\mu{}^{ab} = \omega_\mu{}^{ab}(e) + K_\mu{}^{ab}$$

(a) Polazeći od zakona transformacije

$$\delta_\xi e^a_\mu = -i\kappa \bar{\xi} \gamma^a \psi_\mu \ , \tag{0.25}$$

$$\delta_\xi \psi_\mu = \frac{2}{\kappa} D_\mu \xi \tag{0.26}$$

pokazati da je

$$\delta_\xi \omega_{\mu ab} = -\frac{i\kappa}{2} \Big(\bar{\xi} \gamma_c \psi_{ab} + \bar{\xi} \gamma_b \psi_{ca} - \bar{\xi} \gamma_a \psi_{bc} \Big) \ .$$

(b) Pokazati da je

$$[\delta_\xi, \delta_\eta]\psi_\mu = -\frac{i}{4} \gamma^{bc} \eta e^a_\mu (\bar{\xi} \gamma_c \psi_{ab} + \bar{\xi} \gamma_b \psi_{ca} - \bar{\xi} \gamma_a \psi_{bc}) - (\xi \leftrightarrow \eta) \ .$$

(c) Pokazati da jednačine kretanja gravitina $\gamma^{\nu\rho\sigma} \psi_{\rho\sigma} = 0$, gde je $\psi_{\rho\sigma} = D_\rho \psi_\sigma - D_\sigma \psi_\rho$ u $D = 4$ ima oblik

$$\epsilon^{\mu\nu\rho\sigma} \gamma_5 \gamma_\sigma \psi_{\nu\rho} = 0 \ , \tag{0.27}$$

odnosno

$$\gamma_\sigma \psi_{\nu\rho} + \gamma_\nu \psi_{\rho\sigma} + \gamma_\rho \psi_{\sigma\nu} = 0 \ . \tag{0.28}$$

(d) Pokazati da je

$$[\delta_\xi, \delta_\eta]\psi_\mu = -i(\gamma^b \gamma^c \eta) e^a_\mu \bar{\xi} \gamma_c \psi_{ab} - (\xi \leftrightarrow \eta) \ .$$

Primeniti jednačine kretanja.

(e) Primenom Fircovog identiteta pokazati da je

$$[\delta_\xi, \delta_\eta]\psi_\mu = 2i \bar{\xi} \gamma^\nu (D_\nu \psi_\mu - D_\mu \psi_\nu) \ .$$

Da li je algebra transformacija zatvorena?