KLASIFIKACIJA UV DIJAGRAMA

N_e – broj spolj. e^- linij.
N_γ – ıı – fot – ıı –
P_e – ıı – e^- propagatora
P_γ – broj fot prop
V – broj verteksa
L – broj loopova

$$\text{Dij} \sim \int d^4k_1 \ldots d^4k_L \frac{1}{(\cancel{k}_1 - m)\ldots \quad k_j^2 \ldots k_N^2}$$

SUPERFICIAL DEGREE OF DIVERGENCES $= \mathcal{D} =$

$=$ STEPEN IMPULSA U BROJIOCU – STEP. IMP U IMENIOCU

$$\mathcal{D} = 4L - P_e - 2P_\gamma$$

DIJAGRAM $\sim \Lambda^D$ $\quad$ (Λ – cut off)

$\mathcal{D} = 0$ log diverg
$D = 1$ lin div
$D = 2$ kvad div.
$D < 0$ konvergentan

Ovo je naivno ocekivanje. Uzroci greske
1) dijagram $(D<0)$ ali sadrži DIVERG. PODDIJAGRAM
2) SIMETRIJA UDIJE DIVERG. tj. REDUKUJE divergenciju
3) $D=0$ za dijagr bez prop. i verte.

$$L = P_e + P_\gamma - V + 1$$

$$V = 2P_\gamma + N_\gamma = \frac{1}{2}(2P_e + N_e) \Rightarrow 2P_\gamma - P_e = \frac{N_e}{2} - N_\gamma$$

SVI

$$D = 4(P_e + P_\gamma - V + 1) - P_e - 2P_\gamma$$

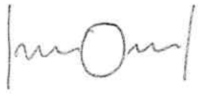
$$= 4(P_e + P_\gamma - 2P_\gamma - N_\gamma + 1) - P_e - 2P_\gamma$$

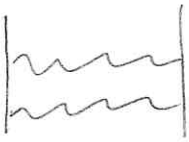
$$= 4 + 3P_e - 6P_\gamma - 4N_\gamma$$

$$\boxed{D = 4 - N_\gamma - \frac{3}{2}N_e}$$

D zavisi od broja externih linija a ne od broja VERTEKSA

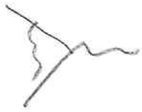
| Greške kod dij-a |

①  $D = 4 - 6 = -2 < 0$ ali on divergira jer sadrži diverg. poddijagram

 $D = -2 < 0$ konačan OK.

②  $D = 2$ ali on ipak divergira log.

③  $D = 0$ konačan (nema loop-a)

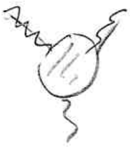
 $D = 0$ div logaritamski OK.

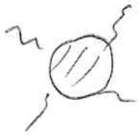
a)  $D = 4$ nebitna (opasna) divergencija jer ne daje doprinos S-matrici

b)  $D = 3$ $= 0$ Furry-jev TEOREM (zadatak

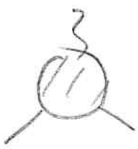
c)  $D = 2$

$\sim (g^{\mu\nu} q^2 - q^\mu q^\nu)\, \Pi(q^2)$ diverg. logaritamski zbog Wardovih id

d)  $D = 1$ $= 0$ Furry-jev T

e)  $D = 0$ ovaj je konačan zbog Ward. id. (dobro jer u QED ne postoji A^4 interakcija

f)  $D = 1$ logaritamski div a ne linearno

g)  $D = 0$ log. div ✓

$= -ie\gamma^\mu \ln \Lambda + \text{finite}$

 $= A_0 + A_1 p\!\!\!/ + A_2 p^2 + \dots$

$$A_n = \frac{1}{n!}\frac{d^n}{dp^n}\left(\;\right)\Big|_{p=0}$$

A_0 = lin

A_1 = log ali nije tako već

A_2 = konačno

 $= \boxed{a_0 m \ln\Lambda + a_1 p\!\!\!/ \ln\Lambda + \text{konačni član}}$

POSTOJI TRI PRIMITIVNO DIV. integrala u QED

POSTOJE <u>ČETIRI</u> diver. broj u QED ($Z_1, Z_2, Z_3, \delta m$)

u d dimenzija

$$D = d + \left(\frac{d-4}{2}\right)V - \frac{d-2}{2}N_\gamma - \frac{d-1}{2}N_e$$

1) za $d<4$ $\Rightarrow$ dijagrami sa više verteksa su postali konvergent

2) za $d>4$ $\Rightarrow$ sa povećanjem broja verteksa raste D i sv. ki dijag. ji diverg. u nekom redu TP

3) za $d=4$ $\Rightarrow$ konačan broj divergencija nezavisan od reda T.P.

$\Rightarrow$ 1) SUPERRENORMALIZABILNO (konačan broj F.d. ji divers.) stupanj diver. opada sa povećanje verteksa

2) NERENORMALIZABILNO — sve amplitude su divergentne i sve je gore u višim redovima TP

3) RENORMALIZABILNO — DIVERGENCIJE prisutne u svim RTP ali postoji konačan broj primitivnih div. stup. div nezavisan od V

φ^n THEORY

$$L = \tfrac{1}{2}(\partial\varphi)^2 - \frac{m^2}{2}\varphi^2 - \frac{\lambda}{n!}\varphi^n$$

N – broj spoljašnjih linija

P – -//- propagatora

$L = P - V + 1$

$nV = N + 2P$

$$\boxed{D = dL - 2P = d + \left[n\left(\frac{d-2}{2}\right) - d\right]V - \left(\frac{d-2}{2}\right)N}$$

$\boxed{d=4}$ ϕ^4 $D = 4 - N$ renormalizabilna

ϕ^6 $D = 4 + 8V - N$ — NERER.

$\boxed{d=3}$ ϕ^6 $D = 3 + (6 \cdot \frac{1}{2} - 3)V - \frac{1}{2}N = 3 - \frac{N}{2}$ REN.

ϕ^4 $D = 3 + (4 \cdot \frac{1}{2} - 4)V - \frac{1}{2}N = 3 - 2V - \frac{N}{2}$ SUPER.

$$D = d - \delta V - \frac{d-2}{2}N$$

δ - kanonske dimenzije od λ $\to$ $[\lambda] = \delta$

1) $\delta = 0 \Rightarrow$ renormalizabilna
2) $\delta < 0 \Rightarrow$ NERENORM.
3) $\delta > 0 \Rightarrow$ SUPER REN.

$\boxed{\text{Weinberg-ov th}}$ Feynmanov dijagrami konvergent ako D za njega i za SVE PODDIJAGRAME < 0

$$S = \frac{1}{8\pi G_N}\int d^4x \sqrt{-g}\, R$$

GRAVITACIJA $[G_N] = -2$

Fermi th. $\sim G_F (\bar\Psi \Gamma \Psi)(\bar\Psi \Gamma \Psi)$ $[G_F] = -2$

$\rightarrow$ OVE TEORIJE nisu renormalizabilne

RENORMALIZACIJA φ^4 TEORIJE

KRENEMO SA GOLOM MASOM I KAPLING KONSTANTOM m_0 λ_0 I SA NEKIM REGULATOROM $\Lambda(\varepsilon)$. NAĐEMO FIZIČKE VELIČINE

$$m = m(m_0, \lambda_0, \varepsilon)$$
$$\lambda = \lambda(m_0, \lambda_0, \varepsilon)$$

TAKOĐE DA BISMO NAŠLI S MATRIČNI element MORAMO ZNATI I Z. ELIMINIŠEMO e_0, m_0 PREKO RENORMALIZOVANE VELIČINE m, λ. AMPLITUDE SU KONAČNE KADA $\Lambda \to \infty$ $(\varepsilon \to 0)$ DUGAČKO U VIŠIM RED. T.P

$$\mathcal{L} = \frac{1}{2}(\partial\phi_0)^2 - \frac{m_0^2}{2}\phi_0^2 - \frac{\lambda_0}{4!}\phi_0^4$$

$$\int d^4x\, e^{ipx} \langle\Omega|T\phi_0(x)\phi_0(0)|\Omega\rangle =$$ 

$=$  $+$  $+$  $+ \dots$

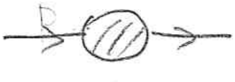
$$= \frac{i}{p^2 - m_0^2 + i\varepsilon} + \left(\frac{i}{p^2 - m_0^2 + i\varepsilon}\right)^2 (-i\Pi(p^2)) + \dots$$

 $= -i\Pi(p^2)$

$\Rightarrow$  $= \dfrac{i}{p^2 - m_0^2 - \Pi(p^2) + i\varepsilon}$

Fizička masa čestice odgovara polu propagatora

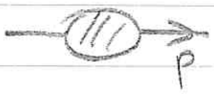
$$m_0^2 + \Pi(m^2) = m^2$$

 $= \dfrac{i}{(p^2 - m^2)(1 - \Pi'(m^2)) - \tilde{\Pi}(p^2)}$

$$= \frac{iZ}{p^2 - m^2 + \frac{\tilde{\Pi}(p^2)}{1 - \Pi'(m^2)}}$$

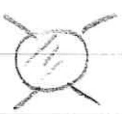
$$\boxed{Z = \frac{1}{1-\Pi'(m^2)}}$$

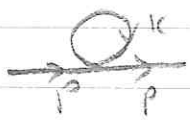
$$\tilde{\Pi}(p^2) = \sum_{i=2}^{\infty} b_i (p^2 - m^2)^i$$

 $= \dfrac{iZ}{p^2 - m^2}$ + OSTATAK

ZA ϕ^4 TEORIJI $\quad D = 4 - N$

Divergentni integrali u ϕ^4 th

	$D = 2$	$\Lambda^2 + p^2 \ln\Lambda$
	$D = 0$	$\sim \ln\Lambda$
	$D = 4$	neopservabilno

$-i\Pi_1(p^2) =$  $= -\dfrac{i\lambda_0}{2}\displaystyle\int \frac{d^4k}{(2\pi)^4}\frac{i}{k^2 - m_0^2}$

1) cut off $\quad -i\Pi_1^c(p^2) = -\dfrac{\lambda_0 i}{2}\displaystyle\int \frac{d^4k_E}{(2\pi)^4}\frac{1}{k_E^2 + m_0^2}$

$$= -\frac{\lambda_0 i}{2}\frac{2\pi^2}{(2\pi)^4}\int_0^{\Lambda^2}\frac{k_E^2\, dk_E^2}{k_E^2 + m_0^2}$$

$$= -\frac{i\lambda_0}{32\pi^2}\left(\Lambda^2 - m_0^2 \ln\frac{\Lambda^2 + m_0^2}{m_0^2}\right) + O(\lambda_0)$$

$$\Rightarrow \quad m^2 = \underbrace{m_0^2}_{\text{beskon}} + \underbrace{\frac{\lambda_0}{32\pi^2}\left(\Lambda^2 - m_0^2 \ln\frac{\Lambda^2 + m_0^2}{m_0^2}\right)}_{\text{BESKONAČNO}} + O(\lambda_0)$$

Konačna fizička masa (RENORMALIZOVANA MASA)

$$\boxed{\delta m^2 \sim \Lambda^2}$$

2) DIMENZIONA REG. $\quad 4 \to D = 4-\varepsilon$

$$-i\Pi_1 = \frac{\lambda_0\mu^\varepsilon}{2}\frac{1}{(2\pi)^D}\int d^Dk\,\frac{1}{k^2-m_0^2}$$

$$= -\frac{\lambda_0\mu^\varepsilon}{2}\,\frac{i\pi^{D/2}}{(2\pi)^D}\,\frac{\Gamma\left(-1+\frac{\varepsilon}{2}\right)}{(m_0^2)^{\frac{\varepsilon}{2}-1}}$$

$$= -\frac{\lambda_0 i}{32\pi^2}\, m_0^2\left(\frac{m_0^2}{4\pi\mu^2}\right)^{-\frac{\varepsilon}{2}}\left(-\frac{2}{\varepsilon}-1+\gamma+O(\varepsilon)\right)$$

$$= \frac{\lambda_0 i}{32\pi^2}\, m_0^2\left(\frac{2}{\varepsilon}+1-\gamma+\ln\frac{m_0^2}{4\pi\mu^2}\right)$$

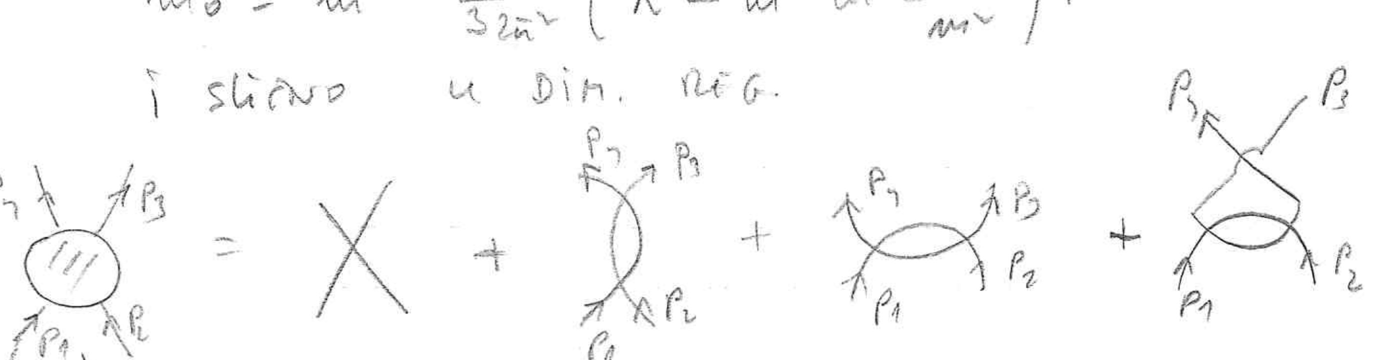
$$\boxed{m^2 = m_0^2 + \underbrace{\frac{\lambda_0}{32\pi^2}\left(-\frac{2}{\varepsilon}-1+\gamma-\ln\frac{m_0^2}{4\pi\mu^2}\right)}_{\text{BESKONAČNA POPRAVKA}} + \dots}$$

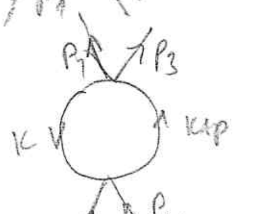
RENORM. MASA (KONAČNO)

SADA INVERTUJEMO OVE RELACIJE

$$m_0^2 = m^2 - \frac{\lambda_0}{32\pi^2}\left(\Lambda^2 - m^2\ln\frac{\Lambda^2+m^2}{m^2}\right)+\dots$$

I SLIČNO U DIM. REG.

$iM \equiv$ 



$$= \frac{(-i\lambda_0)^2}{2}\int\frac{d^4k}{(2\pi)^4}\,\frac{i}{k^2-m_0^2+i\epsilon}\,\frac{i}{(k+p)^2-m_0^2+i\epsilon}$$

$$\equiv (-i\lambda_0)^2\, iV(p^2)$$

$$iV = -\frac{1}{2}\int\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-m_0^2)((k+p)^2-m_0^2)} = -\frac{1}{2}\int_0^1 dx\int\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2+2xkp+xp^2-m_0^2)^2}$$

$$= -\frac{1}{2}\int_0^1 dx\int\frac{d^4l}{(2\pi)^4}\frac{1}{(l^2-\Delta+i\varepsilon)^2}.$$

gde je $\Delta = x^2p^2 - xp^2 + m_0^2$

1) cutoff

$$iV = -\frac{1}{2}\int_0^1 dx\; i\int\frac{d^4l_E}{(2\pi)^4}\frac{1}{(l_E^2+\Delta)^2} = -\frac{1}{2}\int_0^1 dx\,\frac{i2\pi^2}{2(2\pi)^4}\int_0^{\Lambda^2}\frac{l_E^2\,dl_E^2}{(l_E^2+\Delta)^2}$$

$$= -\frac{i}{32\pi^2}\int_0^1 dx\,\ln\frac{\Lambda^2+\Delta}{\Delta}+\dots = -\frac{i}{16\pi^2}\ln\frac{\Lambda}{m_0}$$

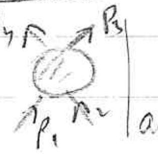
2) dimenziona regularizacija $\quad 4 \to D = 4-\varepsilon$

$$iV = -\frac{1}{2}\mu^{2\varepsilon}\int_0^1 dx\int\frac{d^Dl}{(2\pi)^D}\frac{1}{(l^2-\Delta)^2}$$

$$= -\frac{1}{2}\mu^{2\varepsilon}\frac{i\pi^{D/2}}{(2\pi)^D}\int_0^1 dx\,\frac{\Gamma(\frac{\varepsilon}{2})}{\Delta^{\varepsilon/2}}$$

$$= -\frac{i}{2}\frac{i\mu^{2\varepsilon}}{(4\pi)^{D/2}}\left(\frac{2}{\varepsilon}-\gamma+O(\varepsilon)\right)\int_0^1 dx\left(1-\frac{\varepsilon}{2}\ln(x^2p^2-xp^2+m_0^2)\right)$$

$$= \frac{i\mu^\varepsilon}{32\pi^2}\left(\frac{2}{\varepsilon}-\gamma-\int_0^1\ln\frac{x^2p^2-xp^2+m_0^2}{4\pi\mu^2}\,dx\right)$$

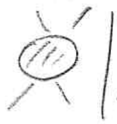
AMPLITUDA 

$\Big|_{\text{amp conn}} = -i\lambda_0\mu^\varepsilon - \lambda_0^2\left(iV(s)+iV(t)+iV(u)\right)$

$$\boxed{\begin{aligned} s &= (p_1+p_2)^2 = p^2 \\ t &= (p_3-p_1)^2 = \\ u &= (p_4-p_1)^2 \end{aligned}}$$

MERIM λ u NEKOJ TAČKI (SVE TAČKE SU podjednako dobre)

Npr $\vec{p}_1=\vec{p}_2=\vec{p}_3=\vec{p}_4=0$ tj. $s=4m^2,\ u=0,\ t=0$

$i\mathcal{M}\Big| =$  $\Big|_{\substack{s=4m^2\\ t=u=0}} = -i\lambda\mu^{\varepsilon}$ (Renorml. uslov)

RENORM. LAMBDA

$$i\mathcal{M} = -i\lambda_0\mu^{\varepsilon} + \frac{i\mu^{\varepsilon}\lambda_0^2}{32\pi^2}\left[\frac{6}{\varepsilon} - 3\gamma - \int_0^1 \ln\frac{s(x^2-x)+m_0^2}{4\pi\mu^2} - \int_0^1 \ln\frac{(x^2-x)t+m_0^2}{4\pi\mu^2}\,dx - \int_0^1 \ln\frac{u(x^2-x)+m_0^2}{4\pi\mu^2}\,dx\right]$$

$$\boxed{\lambda = \lambda_0 - \frac{\lambda_0^2}{32\pi^2}\left[\frac{6}{\varepsilon} - 3\gamma - \underbrace{\int_0^1 dx\,\ln\frac{4m^2(x^2-x)+m_0^2}{4\pi\mu^2}\,dx - 2\int_0^1 \ln\frac{m_0^2}{4\pi\mu^2}\,dx}_{\text{mogu } m_0^2 = m^2 + O(\lambda^2)}\right]}$$

ili

$$\lambda = \lambda_0\left(1 - \frac{3\lambda_0}{16\pi^2}\ln\frac{\Lambda}{m} + \text{kon. termovi}\right)$$

Moramo invertovati ove relacije

$$\boxed{\lambda_0 = \lambda + \frac{\lambda^2\mu^{\varepsilon}}{32\pi^2}\left(\frac{6}{\varepsilon} - 3\gamma - \int_0^1 dx\,\ln\frac{4m^2(x^2-x)+m^2}{4\pi\mu^2} - 2\ln\frac{m^2}{4\pi\mu^2}\right)}$$

λ_0 zavisi od ε (ili Λ) i divergira kod $\varepsilon\to 0$ (tj $\Lambda\to\infty$)

eliminišem λ_0 i m_0 preko λ i m

λ je konačno i ne zavisi od Λ, ε

ali zavisi od tačke u kojoj sam GA DEFINISAO

RADIĆEMO OBRNUTO

BPHZ (Bogoljubov, Parasiuk, Hepp, Zimmermann)

Goli Lagranžijan $\mathcal{L}_0 = \frac{1}{2}(\partial\phi_0)^2 - \frac{m_0^2}{2}\phi_0^2 - \frac{\lambda_0}{4!}\phi_0^4$

$$\int d^4x \langle\Omega|\phi_0(x)\,\phi_0(0)|\Omega\rangle e^{ipx} = \frac{iZ}{p^2-m^2} + \dots$$

$\phi_0 = \sqrt{Z}\,\phi$

→ REN. POLJE

$$\mathcal{L}_0 = \frac{Z}{2}(\partial\phi)^2 - \frac{m_0^2 Z}{2}\phi^2 - \frac{\lambda_0 Z^2}{4!}\phi^4$$

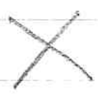
$$\left.\begin{aligned} Z &= 1+\delta Z \\ m_0^2 Z &= m^2 + \delta m^2 \\ \lambda_0 Z^2 &= \mu^\varepsilon(\lambda + \delta\lambda) \end{aligned}\right\}$$

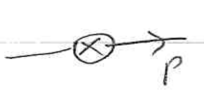
jer želimo da λ bude expanzioni parametar
Razvijamo teoriju oko m, λ
ONI SU KONAČNI!

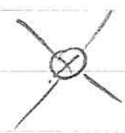
$$\mathcal{L}_0 = \underbrace{\frac{1}{2}(\partial\phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\mu^\varepsilon\lambda}{4!}\phi^4}_{\mathcal{L}_R} + \underbrace{\frac{\delta Z}{2}(\partial\phi)^2 - \frac{\delta m^2}{2}\phi^2 - \frac{\mu^\varepsilon\delta\lambda}{4!}\phi^4}_{\substack{\Delta\mathcal{L} \\ \text{KONTRAČLANOVI}}}$$

Feynman pravila:

 $\frac{i}{p^2-m^2+i\varepsilon}$

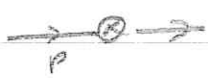
 $-i\lambda\mu^\varepsilon$

 $i(p^2\delta Z - \delta m^2)$

 $-i\delta\lambda\mu^\varepsilon$

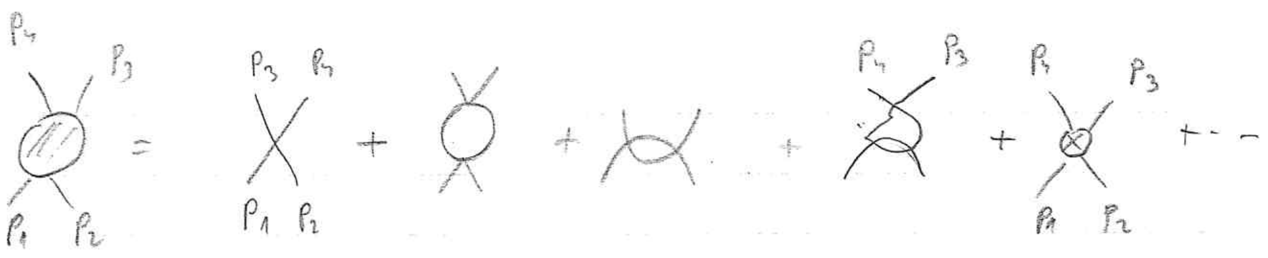
$\mathcal{D} = 4 - N$

Diverg. amplitude

 $=$  $+$  $+ \dots = -i\Pi(p^2)$

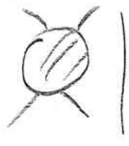
$$= -\frac{i\lambda\mu^\varepsilon}{2}\int\frac{d^4k}{(2\pi)^4}\frac{i}{k^2-m^2} + i(k^2\delta Z - \delta m^2)$$

$$= \frac{i\lambda m^2}{32\pi^2}\left(\frac{2}{\varepsilon} + 1 - \gamma - \ln\frac{m^2}{4\pi\mu^2}\right) + ik^2\delta Z - i\delta m^2$$



KONTRAČLANOVI MORAJU DA APSORBUJU DIVERGENCIJE!
U OVOM MODELU IMAMO 3 DIVERGENTNA BROJA δZ, δm^2, $\delta\lambda$
ZBOG TOGA IMAMO TRI RENORMALIZACIONA USLOVA

$$\left.\begin{array}{l} \Pi(m^2) = 0 \\ \left.\dfrac{d\Pi}{dp^2}\right|_{p^2=m^2} = 0 \end{array}\right\} \Longrightarrow \begin{array}{l} \delta Z = 0 \\ \delta m^2 = \dfrac{m^2\lambda}{32\pi^2}\left(\dfrac{2}{\varepsilon} - 1 - \gamma - \ln\dfrac{m^2}{4\pi\mu^2}\right) \end{array}$$



$$\Big|_{\substack{s=4m^2 \\ t=u=0}} = -i\lambda\mu^{\varepsilon}$$

$$\Downarrow$$

$$\delta\lambda = \frac{\lambda^2}{32\pi^2}\left(\frac{6}{\varepsilon} - 3\gamma - 2\ln\frac{m^2}{4\pi\mu^2} - \int_0^1 dx \ln\frac{4m^2(x^2-x)+m^2}{4\pi\mu^2}\right)$$

RENORMALIZACIJA QED

$$L = -\frac{1}{4} F_{\mu\nu 0} F^{\mu\nu 0} + \bar\Psi_0 (i\not\partial - m_0)\Psi_0 - e_0 \bar\Psi_0 \gamma^\mu \Psi_0 A_\mu^0 - \frac{1}{2\xi_0}(\partial A_0)^2$$

$$\langle \Psi_0 \bar\Psi_0 \rangle \sim \frac{i Z_2}{\not p - m + i\varepsilon} + \dots$$

$$\langle A_\mu^0 A_\nu^0 \rangle = -\frac{i Z_3}{p^2 + i\varepsilon}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) - i\frac{p_\mu p_\nu}{(p^2)^2} + \dots$$

$$\Psi_0 = \sqrt{Z_2}\,\Psi$$
$$A_0^\mu = \sqrt{Z_3}\,A^\mu$$

$$L = -\frac{1}{4} Z_3 F_{\mu\nu}F^{\mu\nu} + Z_2 \bar\Psi (i\not\partial - m_0)\Psi - e_0 Z_2 \sqrt{Z_3}\, \bar\Psi \gamma^\mu \Psi A_\mu - \frac{Z_3}{2\xi_0}(\partial A)^2$$

$$Z_3 = 1 + \delta_3$$
$$Z_2 = 1 + \delta_2$$
$$Z_2 m_0 = m + \delta m$$
$$e_0 Z_2 \sqrt{Z_3} = e Z_1 \mu^{\varepsilon/2} = e\mu^{\frac{\varepsilon}{2}}(1+\delta_1)$$

$$\frac{Z_\xi}{\xi_0} = \frac{1+\delta_\xi}{\xi}$$

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar\Psi(i\not\partial - m)\Psi - e\mu^{\frac{\varepsilon}{2}}\bar\Psi\gamma^\mu\Psi A_\mu - \frac{1}{2\xi}(\partial A)^2$$
$$\underbrace{- \frac{\delta_3}{4}F_{\mu\nu}F^{\mu\nu} + \delta_2 \bar\Psi i\not\partial\Psi - \delta_m \bar\Psi\Psi - e\mu^{\varepsilon/2}\delta_1 \bar\Psi \not A \Psi - \frac{\delta_\xi}{\xi}(\partial A)^2}_{\text{kontračlanovi}}$$

F. pravila

 $\frac{-i g_{\mu\nu}}{p^2 + i\varepsilon}$ ($\xi = 0$ Feynman-ov gauge)

 $\frac{i}{\not p - m + i\varepsilon}$

 $-ie\gamma^\mu \mu^{\varepsilon/2}$

 $-i(g^{\mu\nu}q^2 - q^\mu q^\nu)\delta_3$

 $i(\not p\,\delta_2 - \delta_m)$

 $-ie\mu^{\varepsilon/2}\delta_1\gamma^\mu$

DIVERG. AMPL.

 $= i\Pi_{\mu\nu}(p)$

 $= -i\Sigma(p)$

 $= -ie\Gamma^\mu(p,p',q)$

Renormalizacioni uslovi:

- $\Sigma(\not p = m) = 0$
- $\frac{d\Sigma}{d\not p}\Big|_m = 0$
- $-ie\Gamma^\mu(p,p,0) = -ie\gamma^\mu$

$-i\Sigma(p) =$  $+$ 

RAČUNAM $-i\Sigma_2$ DIMENZIONA regularizacija $(\xi = 1)$

$$-i\Sigma_2(p) = (-ie)^2\mu^\varepsilon\int\frac{d^Dk}{(2\pi)^D}\,\frac{\gamma^\mu(\not k + m)\gamma_\mu}{(k^2-m^2+i\varepsilon)\,((p-k)^2-\mu^2+i\varepsilon)}$$

→ iC PARAMETRI

$$= -e^2\mu^\varepsilon\int_0^1 dx\int\frac{d^Dl}{(2\pi)^D}\,\frac{-(D-2)\not p\,x + mD}{(l^2-\Delta+i\varepsilon)^2}$$

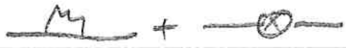
GDE je $\Delta = m^2(1-x) + x\mu^2 - x(1-x)p^2$

$$-i\Sigma_2(p) = -e^2\mu^\varepsilon\int_0^1 dx\,\frac{i\pi^{D/2}}{(2\pi)^D\Delta^{\varepsilon/2}}\,\Gamma\!\left(\frac{\varepsilon}{2}\right)\left(mD - (D-2)x\not p\right)$$

$$\frac{\pi^{D/2}\mu^\varepsilon}{(2\pi)^D} = \frac{\mu^\varepsilon}{(4\pi)^{D/2}} = \frac{1}{16\pi^2}\left(4\pi\mu^2\right)^{\varepsilon/2}$$

$$-i\Sigma_2(p) = -\frac{ie^2}{16\pi^2}\int_0^1 dx\left(1 - \frac{\varepsilon}{2}\ln\frac{\Delta}{4\pi\mu^2}\right)\left(\frac{2}{\varepsilon} + \dots\right)\left((-2+\varepsilon)\not p\,x + (4-\varepsilon)m\right)$$

$$= -\frac{ie^2}{16\pi^2}\,\frac{2}{\varepsilon}\left(-\not p + 4m\right) + \text{Finite part}$$

$-i\Sigma(p) =$ 

$$= -\frac{ie^2}{8\pi^2\varepsilon}(-\not p + 4m) + \text{kon. deo} + i(\not p\,\delta_2 - \delta m)$$

$$\left.\frac{d\Sigma}{d\not p}\right|_m = \frac{ie^2}{8\pi^2\varepsilon} + i\delta_2 = 0 \Rightarrow \boxed{\delta_2 = -\frac{e^2}{8\pi^2\varepsilon} + \text{K.D}}$$

$$\Sigma(\not p = m) = -\frac{ie^2}{8\pi^2\varepsilon}3m - \frac{ie^2}{8\pi^2\varepsilon} + \text{K.D} - i\delta m \Rightarrow$$

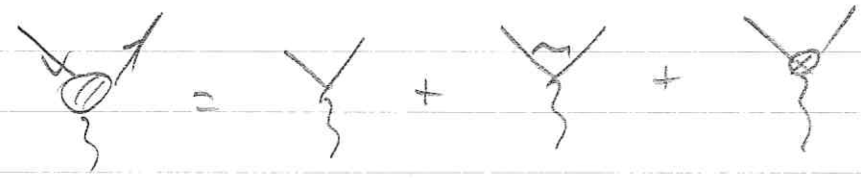
$$\Rightarrow \boxed{\delta m = -\frac{e^2 m}{2\pi^2\varepsilon} + \text{kon. deo}}$$

$\circ\ i\Pi^{\mu\nu} =$ 

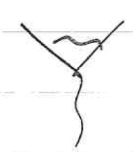
$$= (q^\mu q^\nu - q^2 g^{\mu\nu})\frac{8ie^2}{(4\pi)^2}\frac{1}{3\varepsilon} + \text{kon. deo} + i(q^\mu q^\nu - q^2 g^{\mu\nu})\delta_3$$

$$\Pi_2(q^2=0) = -\frac{8e^2}{(4\pi)^2}\frac{1}{3\varepsilon} - \delta_3 + \text{K.D} = 0$$

$$\Rightarrow \boxed{\delta_3 = -\frac{e^2}{10\pi^2\varepsilon} + \text{kon. deo}}$$

$\circ$ 

$$-ie\gamma^\mu\mu^{\varepsilon/2} + ie\,\delta\Gamma^\mu(0) - ie\gamma^\mu\delta_1\mu^{\varepsilon/2} = -ie\gamma^\mu\mu^{\varepsilon/2}$$

 $= \mu^{\frac{\varepsilon}{2}}\frac{e^2}{8\pi^2\varepsilon}\gamma^\mu + \text{div deo}$

$$\Rightarrow \boxed{\delta_1 = -\frac{e^2}{8\pi^2\varepsilon} + \text{K.D}}$$

Dakle:

$$Z_1 = 1 - \frac{e^2}{8\pi^2\varepsilon} + \dots$$
$$Z_2 = 1 - \frac{e^2}{8\pi^2\varepsilon} + \dots \quad \Big\} \quad Z_1 = Z_2 \checkmark$$
$$Z_3 = 1 - \frac{e^2}{6\pi^2\varepsilon} + \dots$$
$$\delta m = - \frac{e^2 m}{8\pi^2\varepsilon} + \dots$$
$$e = \sqrt{Z_3}\, e_0 \checkmark$$

LAMBOV POMAK

RAZMATRAMO RASEJANJE e^- NA Kulonovom potencijalu

OTPADAJU ZBOG JED. KRET.

$$iM = \tilde{A}_\mu(q)\bar{u}(p')\Big\{-ie\gamma^\mu - ie\gamma^\mu \frac{i}{\not p - m}(-i\Sigma(p)) + (-i\Sigma(p'))\frac{i}{\not p' - m}(-ie\gamma^\mu)$$
$$-ie\delta\Gamma^\mu - ie\gamma^\mu\delta_1 - ie\gamma_\rho \frac{-ig^{\rho\nu}}{q^2+i\varepsilon}\, i(g^\mu_\nu q^2 - q_\nu q^\mu)\underbrace{\left(\Pi_2(q^2) - \delta_3\right)}_{\hat{\Pi}_2(q^2)}\Big\}u(p)$$

$$\longrightarrow \bar{u}(p')(-ie\gamma^\mu)\left(1 + \hat{\Pi}_2(q^2)\right)u(p)\,\tilde{A}_\mu(q)$$

$$\hat{\Pi}_2(q) = -\frac{2\alpha}{\pi}\int_0^1 dx\, x(1-x)\ln\frac{m^2}{m^2 - x(1-x)q^2}$$

$$q^0 = E_f - E_i = 0 \qquad \vec{q} = \vec{p}\,' - \vec{p} \quad \text{(TRANSFER IMP.)}$$

$$|q^2| = |(q^0)^2 - \vec{q}^{\,2}| = |-\vec{q}^{\,2}|$$

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PRIMENIĆEMO NR APROKSIMACIJU $|-\vec{q}^2| \ll m^2$

$$\Pi_2(q^2) = +\frac{2\alpha}{\pi}\int_0^1 dx\, x(1-x)\ \ln\left|1 - x(1-x)\frac{q^2}{m^2}\right|$$

$$\cong \frac{2\alpha}{\pi}\ \frac{\vec{q}^{\,2}}{30m^2}$$

Kulonov potencijal je modifikovan.

$$\tilde{A}_0(q) = \left(1 + \hat{\Pi}_2(q^2)\right)\tilde{A}_0(q) = \left(1 - \frac{\alpha\,\vec{q}^{\,2}}{15\pi^2 m}\right)\tilde{A}_0(q^2)$$

$$A_0(x) = \int\frac{d^4q}{(2\pi)^4}\ e^{-iqx}\ \tilde{A}_0(q)$$

$$= \int\frac{dq_0\, d^3q}{(2\pi)^4}\ e^{iq_0 t + i\vec{q}\vec{r}}\ 2\pi\,\delta(q^0)\,(-Ze)\left(\frac{1}{\vec{q}^{\,2}} + \frac{\alpha}{15\pi m^2}\right)$$

$$= -\frac{Ze}{4\pi r} - \frac{Ze\alpha}{15\pi m^2}\,\delta^{(3)}(\vec{r})$$

$Z = 1$

$$\Delta E = \int d^3r\,|\psi(\vec{r})|^2\ \frac{-4\alpha^2}{15m^2}\,\delta^{(3)}(\vec{x}) = -\frac{4\alpha^2}{15m^2}\,|\psi(0)|^2$$

ZA 2S STANJE

$$\Delta E = -\frac{\alpha^5 m}{30\pi} = -1{,}123\cdot 10^{-7}\,eV$$

Lambov p.