FIELD STRENGTH RENORMALIZATION

RAZMATRAMO SKALARNO polje

ZANIMA NAS ANALITIČKA STRUKTURA $\langle\Omega| T\phi(x)\phi(y)|\Omega\rangle$ U int-TP.

$$|\Omega\rangle = |0\rangle + \sum_n \frac{\langle n|V|0\rangle}{E_n - E_0}|n\rangle + \dots \qquad (|0\rangle \neq |\Omega\rangle)$$

$[H, \vec{P}] \Rightarrow$

$\{|\lambda_0\rangle\} \rightarrow$ svojstvena stanja H sa imp. $\vec{p} \Rightarrow$

$$\vec{P}|\lambda_0\rangle = 0$$

$\{|\lambda_p\rangle\} \rightarrow$ svojstvena stanja H dobijena sa BOOSTOM iz $|\lambda_0\rangle$

$$E_\lambda = \sqrt{\vec{p}^2 + m_\lambda^2}$$

(STANJA SU REL. norm. $\langle\vec{p}|\vec{q}\rangle = 2E_p(2\pi)^3\delta^{(3)}(p-q)$)

- SLOB. T.P. $\quad (\mathbb{1})_{1-particle} = \int \frac{dp}{(2\pi)^3}\frac{1}{2E_p}|\vec{p}\rangle\langle\vec{p}|$
- int. $\quad \mathbb{1} = |\Omega\rangle\langle\Omega| + \sum_{\lambda_0}\int\frac{dp}{(2\pi)^3}\frac{1}{2E_p(\lambda)}|\lambda_p\rangle\langle\lambda_p|$

(oznaka $|\lambda_p\rangle \equiv |p, \lambda\rangle$ $\hookrightarrow$ drugi kvantni brojevi, mase, relativni impuls, $\vec{L}$

$W(x,y) = \langle\Omega|\phi(x)\phi(y)|\Omega\rangle$ (Wightman-ova fja)

$$= \sum_\lambda \int\frac{dp}{(2\pi)^3}\frac{1}{2E_p(\lambda)}\langle\Omega|\phi(x)|\lambda_p\rangle\langle\lambda_p|\phi(y)|\Omega\rangle + \cancel{\langle\Omega|\phi(x)|\Omega\rangle\langle\Omega|\phi(y)|\Omega\rangle}$$

$$\langle\Omega|\phi(x)|\lambda_p\rangle = \langle\Omega|e^{iPx}\phi(0)e^{-iPx}|\lambda_p\rangle$$
$$= e^{-ipx}\langle\Omega|\phi(0)|\lambda_p\rangle\Big|_{p^0=E_p}$$
$$= e^{-ip_{\mu}x^{\mu}}\langle\Omega|U_{0p}^{-1}U_{0p}\phi(0)U_{0\to p}|\lambda_0\rangle$$

$\hookrightarrow$ BOOST $0\to p$

$$= e^{-ip\cdot x}\langle\Omega|\phi(0)|\lambda_0\rangle\Big|_{p^0=E_p}$$

$$W(x,y) = \langle\Omega|\phi(x)\phi(y)|\Omega\rangle = \sum_\lambda\int\frac{d\vec{p}}{(2\pi)^3}\frac{1}{2E_p(\lambda)}e^{-ip(x-y)}|\langle\lambda_0|\phi(0)|\Omega\rangle|^2\Big|_{p_0=E_p}$$

(ZA $x^0 > y^0$)

$\int\frac{d^4p}{(2\pi)^4}\delta$

$$W(x-y) \overset{x^0>y^0}{=} \sum_\lambda \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2-m_\lambda^2+i\varepsilon} e^{-ip(x-y)} \left|\langle \lambda_0 | \phi(0) | \Omega\rangle\right|^2$$

Analogno važi za $y^0 > x^0 \implies$

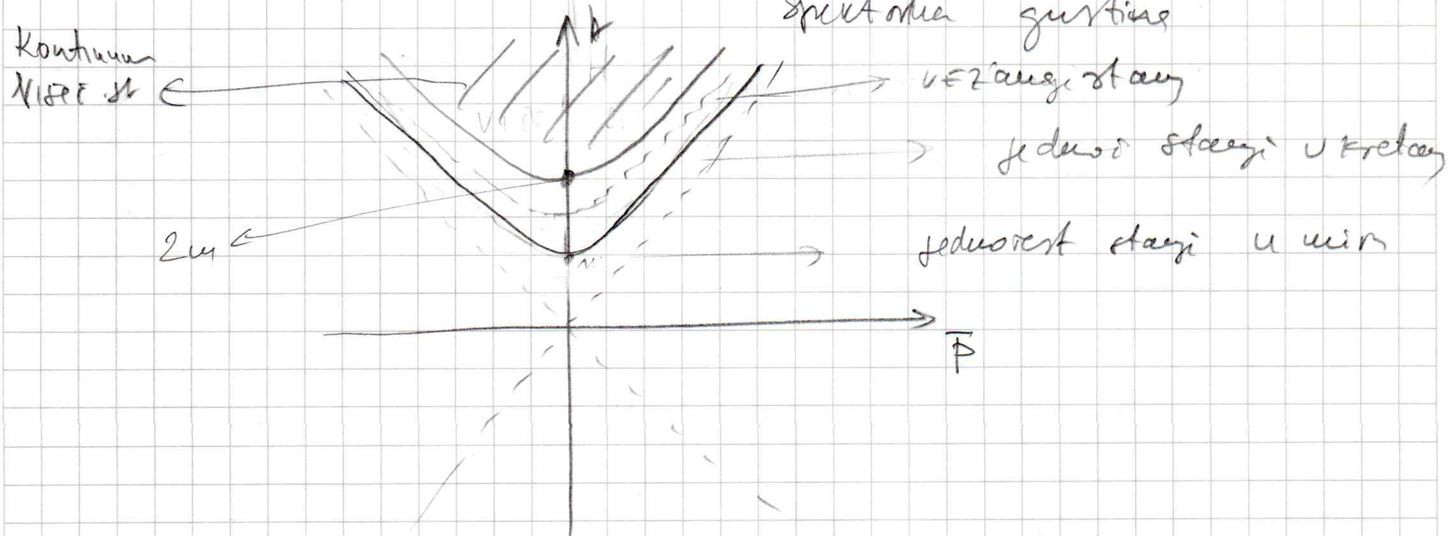
$$\boxed{\langle \Omega | T\phi(x)\phi(y) | \Omega\rangle} = \sum_\lambda i\Delta_F(x-y, m_\lambda^2) \left|\langle \lambda_0 | \phi(0) | \Omega\rangle\right|^2$$

$$= \int_0^\infty ds \sum_\lambda \delta(s-m_\lambda^2)\, i\Delta_F(x-y, s) \left|\langle \lambda_0 | \phi(0) | \Omega\rangle\right|^2$$

$$= \boxed{\int_0^\infty \frac{ds}{2\pi}\, \rho(s)\, i\Delta_F(x-y, s)} \quad \text{Källen-Lehmann rep}$$

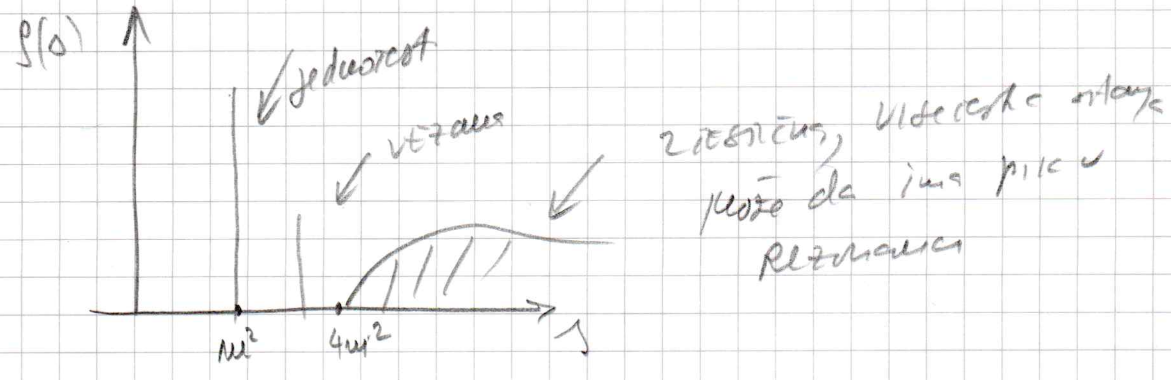
gde je $\rho(s) = \sum_\lambda 2\pi\, \delta(s-m_\lambda^2) \left|\langle \lambda_0 | \phi(0) | \Omega\rangle\right|^2$

Spektralna gustina



$|\Omega\rangle$ - osn. stanje

Energija višečest. stanja, $E = \sqrt{\vec{p}^2+m_\lambda^2}$ $\quad m_\lambda > 2m$



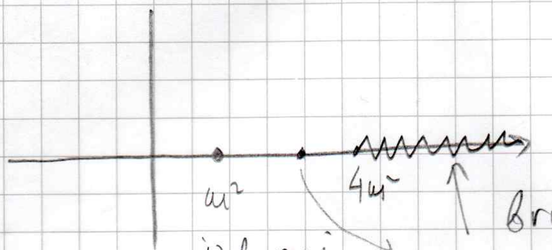
$$\rho(s) = 2\pi\,\delta(s-m^2)\,Z + \overset{\sigma(s)}{\text{OSTATAK}}$$

m^2: PRAVA MASA ČEST. (ne m_0)

Z → renormalizaciona konstanta → $|\langle\lambda_0|\phi(0)|\Omega\rangle|^2$

$$\int d^4x\, e^{ipx}\langle\Omega|T\,\phi(x)\,\phi(0)|\Omega\rangle = \int_0^\infty \frac{ds}{2\pi}\,\rho(s)\,\frac{i}{p^2-s^2+i\varepsilon}$$

$$= \underbrace{\frac{iZ}{p^2-m^2+i\varepsilon}}_{\substack{\text{izolovani pol}\\ \text{rezidum } Z}} + \underbrace{\int_{\sim 4m^2}^{\infty}\frac{ds}{2\pi}\,\rho(s)\,\frac{i}{p^2-s^2+i\varepsilon}}_{\text{OSTATAK}}$$



- SLOBODNA TEORIJA

$$\int d^4x\, e^{ipx}\langle\Omega|T\,\phi(x)\,\phi(0)|\Omega\rangle = \frac{i}{p^2-m^2+i\varepsilon}$$

$Z = |\langle\lambda_0|\phi(0)|\Omega\rangle|^2$ → VEROVATNOĆA DA $\phi(0)$ kreira jednočest. stanje iz vakuuma

U slob. TEOR. polja $\phi(0)$ ne može da kreira višečestično stanje iz vakuuma

Zato $Z=1$ u slob. T.P., a

$Z<1$ u interakcionoj teoriji

$$\boxed{\phi_{phys} = \sqrt{Z}\,\phi}$$

- SLUČAJ SPINORA

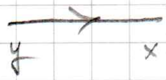
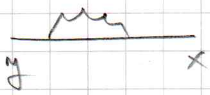
$$\int d^4x\, e^{ipx}\langle\Omega|T\,\psi(x)\,\bar\psi(0)|\Omega\rangle = \frac{iZ_2}{p^2-m^2+i\varepsilon}\sum u_r(p)\,\bar u_r(p) \text{ ozn.} + \text{OSTATAK}$$

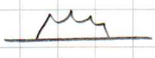
$$= iZ_2\,\frac{i}{\not p - m + i\varepsilon} + \dots$$

Z_2 – VEROVATNOĆA DA KV. POLJE kreira ili anihilira 1-čest stanje

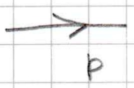
$$\langle p, s | \psi(x) | \Omega \rangle = v_s(p)\, e^{-ipx}, \sqrt{Z_2}$$
$$\langle p, s | \bar{\psi}(x) | \Omega \rangle = u_s(p)\, e^{ipx} \sqrt{Z_2}$$

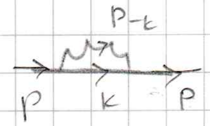
SELF – ENERGY OF ELECTRON

$\langle \Omega | T\, \psi(x) \bar{\psi}(y) | \Omega \rangle =$  $+$  $+ \cdots$

$\int d^4x\, e^{ipx} \langle \Omega | \psi(x) \bar{\psi}(y) | \Omega \rangle =$  $+$ 

↑ self energy

 $= \dfrac{i}{\not{p} - m_0 + i\varepsilon}$

 $= \dfrac{i}{\not{p} - m_0 + i\varepsilon} \left(-i \Sigma_2(p) \right) \dfrac{i}{\not{p} - m_0 + i\varepsilon}$

$$-i\Sigma_2(p) = (-ie)^2 \int \frac{d^4k}{(2\pi)^4} \gamma^\mu \frac{i(\not{k} + m_0)}{k^2 - m_0^2 + i\varepsilon} \gamma_\mu \frac{-i}{(p-k)^2 - \mu^2 + i\varepsilon}$$

↖ IR regulator

$$\text{Fp:} \quad \frac{1}{k^2 - m_0^2 + i\varepsilon} \, \frac{1}{(p-k)^2 + i\varepsilon + i\varepsilon} = \int_0^1 dx \frac{1}{\left(k^2 - m_0^2 + x(p^2 - 2pk - \mu^2 + m_0^2) + i\varepsilon \right)^2}$$

$$= \int_0^1 dx \frac{1}{\Big[\underbrace{(k - xp)}_{l}{}^2 - m_0^2 + x(p^2 + m_0^2 - \mu^2) - x^2 p^2 + i\varepsilon \Big]^2}$$

$$-i\Sigma_2(p) = -e^2 \int_0^1 dx \int \frac{d^4 l}{(2\pi)^4} \frac{-2(\not{l} + x\not{p}) + 4m_0}{[l^2 - \Delta + i\varepsilon]^2}$$

$$= -e^2 \int_0^1 dx \int \frac{d^4 l}{(2\pi)^4} \frac{-2\not{p}x + 4m_0}{[l^2 - \Delta + i\varepsilon]^2} \quad \sim \int \frac{dl}{l} = \ln \Lambda$$

$$\Delta = m_0^2 (1-x) + x\mu^2 - x(1-x)p^2$$

$$-i\Sigma_2(p) = -e^2\int_0^1 dx\,(-2x\not{p}+4m_0)\underbrace{\int\frac{d^4l}{(2\pi)^4}\frac{1}{[l^2-\Delta+i\varepsilon]^2}}_{I}$$

① **cutoff**

$$I = \frac{2\pi^2}{(2\pi)^4}\, i\int_0^\Lambda\frac{l_E^3\,dl_E}{(l_E^2+\Delta)^2}$$

$$= \frac{i}{2\cdot 8\pi^2}\int_0^{\Lambda^2}\frac{l_E^2\,dl_E^2}{(l_E^2+\Delta)^2}$$

$$= \frac{i}{16\pi^2}\left[\ln(l_E^2+\Delta)\Big|_0^{\Lambda^2} + \Delta\frac{1}{l_E^2+\Delta}\Big|_0^{\Lambda^2}\right]$$

$$= \frac{i}{16\pi^2}\ln\frac{\Lambda^2}{\Delta} +$$

$$-i\Sigma_2(p) = -e^2\int_0^1 dx\,(-2x\not{p}+4m_0)\frac{i}{16\pi^2}\ln\frac{\Lambda^2}{\Delta}$$

② **Pauli - Vilars-ova regularizacija**

$$\frac{1}{(p-k)^2-\mu^2+i\varepsilon} \to \frac{1}{(p-k)^2-\mu^2+i\varepsilon} - \frac{1}{(p-k)^2-\Lambda^2+i\varepsilon}$$

$$-i\Sigma_2(p) = -e^2\int_0^1 dx\int\frac{d^4l}{(2\pi)^4}(-2x\not{p}+4m_0)\left[\frac{1}{(l^2-\Delta+i\varepsilon)^2} - \frac{1}{(l^2-\Delta_\Lambda+i\varepsilon)^2}\right]$$

$$= -e^2\int_0^1 dx\,(-2x\not{p}+4m_0)\,\times$$

$$\times\frac{i}{(4\pi)^2}\left[\ln\frac{l_E^2+\Delta}{l_E^2+\Delta_\Lambda} + \frac{\Delta}{l_E^2+\Delta} - \frac{\Delta_\Lambda}{l_E^2+\Delta_\Lambda}\right]\Bigg|_0^\infty$$

$$\Delta_\Lambda = -x(1-x)p^2 + x\Lambda^2 + (1-x)m_0^2 \xrightarrow[\Lambda\to\infty]{} x\Lambda^2$$

$$-i\Sigma_2(p) = -\frac{ie^2}{(4\pi)^2}\int_0^1 dx\,(4m_0-2x\not{p})\ln\frac{\Delta_\Lambda}{\Delta}$$

$$= \frac{\alpha}{2\pi}i\int_0^1 dx\,(2m_0-x\not{p})\ln\frac{x\Lambda^2}{-x(1-x)p^2+(1-x)m_0^2+x\mu^2}$$

$W = \ln z$ ima branch point at $z=0$ and branch cut duž ~~pos~~ neg. dela x ose

$$(1-x)\,m_0^2 - (1-x)\,x\,p^2 + x\mu^2 = 0$$

$$p^2 = \frac{(1-x)\,m_0^2 + x\mu^2}{x(1-x)} = \frac{m_0^2}{x} + \frac{\mu^2}{1-x}$$

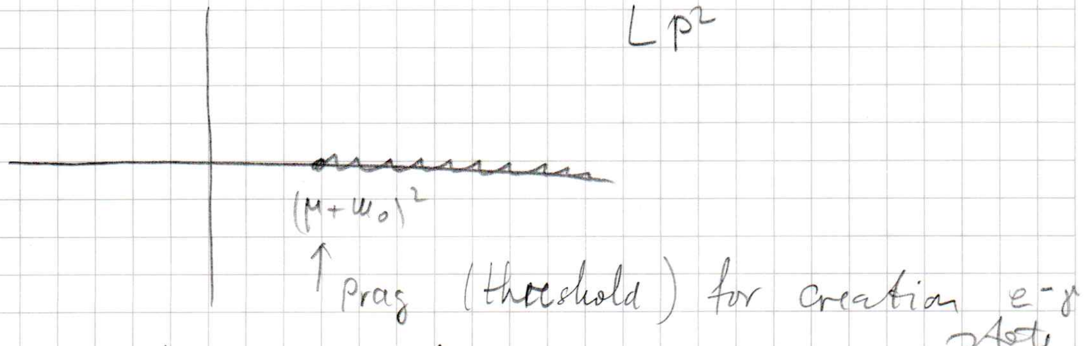
$$\frac{\partial p^2}{\partial x} = -\frac{m_0^2}{x^2} + \frac{\mu^2}{(1-x)^2} = 0$$

$$m_0^2(1-x)^2 - \mu^2 x^2 = 0$$

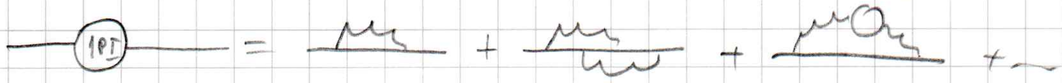
$$\frac{1}{x} - 1 = + \frac{\mu}{m_0} \qquad \text{jer je } 0 < x < 1$$

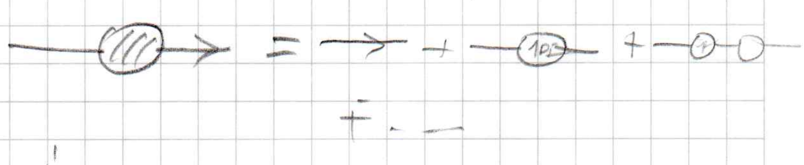
$$p^2 = m_0^2\left(1 + \frac{\mu}{m_0}\right) + \frac{\mu^2}{\mu/m_0}\left(1 + \frac{\mu}{m_0}\right)$$

$$= m_0^2 + 2\mu m_0 + \mu^2 = (m_0+\mu)^2$$



• Pol propagatora treba da bude u $p^2 = m^2$

$-i\,\Sigma(p) =$ 

$\int d^4x\, \langle\Omega|T\,\psi(x)\,\bar\psi(0)|\Omega\rangle\, e^{ipx} =$ 

$$= \frac{i}{\not p - m_0 + i\varepsilon} + \frac{i}{\not p - m_0}\left(-i\,\Sigma(p)\right)\frac{i}{\not p - m_0} + \dots$$

$$= \frac{i}{\not p - m_0}\left(1 + \frac{\Sigma(p)}{\not p - m_0} + \left(\frac{\Sigma(p)}{\not p - m_0}\right)^2 + \dots\right)$$

$$= \frac{i}{\not p - m_0}\;\frac{1}{1 - \frac{\Sigma(p)}{\not p - m_0}} = \frac{i}{\not p - m_0 - \Sigma(p)}$$

LOKALIZACIJA POLA $\not p - m_0 - \Sigma(\not p)\Big|_{\not p = m} = 0$

FIZIČKA MASA

BLIZU POLA $\Sigma(\not p) = A + B(\not p - m) + (\not p - m)\Sigma_c$

$$\not p - m_0 - \Sigma(\not p) = \not p - m_0 - \Sigma(m) - (\not p - m)\frac{d\Sigma}{d\not p}\Big|_m - (\not p - m)\Sigma_c$$

$$= (\not p - m)\Big(1 - \frac{d\Sigma}{d\not p}\Big|_m - \underbrace{\Sigma_c}_{\mathcal{O}(\not p - m)}\Big)$$

$$m = m_0 + \underbrace{\Sigma(\not p = m)}_{= \delta m = A}$$ РЕНОРМАЛИЗОВАНА МАСА

$$\frac{i}{\not p - m_0 - \Sigma(\not p)} = \frac{i}{(\not p - m)\Big(1 - \frac{d\Sigma}{d\not p}\Big|_m - \Sigma_c\Big)}$$

$$= \frac{i Z_2}{\not p - m} + \ldots$$

gdje je $\boxed{Z_2 = \frac{1}{1 - \frac{d\Sigma}{d\not p}\Big|_m}}$

$$\delta m = m - m_0 \simeq \Sigma_2(\not p = m) \cong \Sigma_2(\not p = m_0)$$

$$= \underline{\frac{\alpha m_0}{2\pi} \cdot \frac{3}{2} \ln\frac{\Lambda^2}{m_0^2}}$$ РЕНОРМ. МАСЕ

(SLABIJA DIV. nego u KLAS ELD)

$$\boxed{m = m_0 + \underbrace{\frac{3\alpha}{4\pi} m_0 \ln\frac{\Lambda^2}{m_0^2}}_{\to \infty}}$$ → РЕН МАСА

$$Z_2 = 1 + \delta Z_2$$

$$\delta Z_2 = \frac{d\Sigma_2}{d\not p}\Big|_m = \frac{\alpha}{2\pi}\int_0^1 dx\Big[-x\ln\frac{x\Lambda^2}{m^2(1-x)^2 + x\mu^2}$$

$$+ m(2-x)\frac{2m\,x(1-x)}{m^2(1-x)^2 + x\mu^2}\Big] \sim \ln\Lambda$$

POKAZATI $\underline{\underline{\delta F_1(0) = -\delta Z_2}}$

БЕЗ ф.? перидук

18a

$$\delta F_1(0) = \frac{\alpha}{2\pi} \int_0^1 dx\,dy\,dz\; \delta(x+y+z-1) \left[\ln \frac{z\Lambda^2}{(1-z)^2 m^2 + z\mu^2} + \frac{(1-4z+z^2)m^2}{(1-z)^2 m^2 + z\mu^2} \right]$$

$$= \frac{\alpha}{2\pi} \int_0^1 dz\,(1-z) \left[\ln \frac{z\Lambda^2}{(1-z)^2 m^2 + \mu^2 z} + \frac{(1-4z+z^2)m^2}{(1-z)^2 m^2 + z\mu^2} \right]$$

$$\delta F_1(0) + \delta Z_2 =$$

$$\frac{\alpha}{2\pi} \int_0^1 dz \Big[\underbrace{(1-2z)}_{dv} \underbrace{\ln \frac{z\Lambda^2}{(1-z)^2 m^2 + \mu^2 z}}_{u} + 2(2-z) \frac{z(1-z)m^2}{(1-z)^2 m^2 + z\mu^2} + (1-z) \frac{(1-4z+z^2)m^2}{(1-z)^2 m^2 + z\mu^2} \Big]$$

$uv - \int v\,du$

$$\ln \frac{z\Lambda^2}{(1-z)^2 m^2 + \mu^2 z} \cdot (z - z^2) \Big|_0^1$$

$$- \int (z - z^2) \left(\frac{1}{z} - \frac{\mu^2 - 2(1-z)m^2}{(1-z)^2 m^2 + \mu^2 z} \right)$$

$=$

$$\boxed{\delta F_1(0) + \delta Z_2 = 0}$$

$$- \int \cancel{z}(1-z) \frac{(1-z)^2 m^2 + \cancel{\mu^2 z} - \cancel{\mu^2 z} + 2z(1-z)m^2}{\cancel{z}\left[(1-z)^2 m^2 + \mu^2 z\right]}$$

$$= - \int (1-z) \frac{1 - z^2}{} m^2$$

$$4z - 2z^2 + 1 - 4z + z^2$$

$$(1-z) \frac{2(2-z)z m^2 + (1-4z+z^2)m^2}{(1-z)^2 m^2 + \mu^2 z}$$

$\boxed{\text{OK}}$

dobija se