RENORMALIZACIONE ŠEME U φ^4 TEORIJI

$$\tilde{\Gamma}^{(2)} = p^2 - m^2 - \Pi(p^2) = (1+\delta z)\, p^2 - m^2 + \frac{\lambda m^2}{32\pi^2}\left(\frac{2}{\varepsilon} + 1 - \gamma - \ln\frac{m^2}{4\pi\mu^2}\right) - \delta m^2$$

$$\tilde{\Gamma}^{(4)} = -i\lambda\mu^{\varepsilon} + \frac{i\mu^{\varepsilon}\lambda^2}{32\pi^2}\left\{\frac{6}{\varepsilon} - 3\gamma + 3\ln\frac{4\pi\mu^2}{m^2} - \int_0^1 dx \ln\left(1 + \frac{s}{m^2}(x^2-x)\right)\right.$$
$$\left. - \int_0^1 dx \ln\left(1 + \frac{u}{m^2}(x^2-x)\right) - \int_0^1 dx \ln\left(1 + \frac{t}{m^2}(x^2-x)\right)\right\} - i\delta\lambda\mu^{\varepsilon}$$

UVODIMO OZNAKU: $A(s, m^2) = \int_0^1 dx \ln\left(1 + \frac{s}{m^2}(x^2 - m^2)\right)$

PARAMETRI KONTRAČLAN. δm^2, δz, $\delta\lambda$ MORAJU DA APSORBUJU DIVERGENCIJE

① ON-SHELL R. ŠEMA

$\tilde{\Gamma}^2\big|_{p^2=m^2} = 0 \quad \Rightarrow \quad \boxed{\delta m^2 = \frac{\lambda m^2}{32\pi^2}\left(\frac{2}{\varepsilon} + 1 - \gamma - \ln\frac{m^2}{4\pi\mu^2}\right)}$

$\frac{\partial \tilde{\Gamma}^2}{\partial p^2}\Big|_{p^2=m^2} = 1 \quad \Rightarrow \quad \boxed{\delta z = 1}$

$\tilde{\Gamma}^4\Big|_{s=t=u=\frac{4m^2}{3}} = -i\lambda\mu^{\varepsilon}$

$\frac{4m^2}{3} \rightarrow$ SIMETRIČNA TAČKA

$$\Rightarrow \boxed{\delta\lambda = \frac{3\lambda^2}{32\pi^2}\left(\frac{2}{\varepsilon} - \gamma + \ln\frac{4\pi\mu^2}{m^2} - A\left(\frac{4m^2}{3}, m^2\right)\right)}$$

DAKLE, RENORMALIZOVANE 1PI FUNKCIJE SU

$$\tilde{\Gamma}^{(2)} = p^2 - m^2$$

$$\tilde{\Gamma}^{(4)} = i\mu^{\varepsilon}\left[-\lambda + \frac{\lambda^2}{32\pi^2}\left(A\left(\frac{4m^2}{3}, m^2\right) - A(s, m^2) - A(u, m^2) - A(t, m^2)\right.\right.$$

ONE SU KONAČNE KADA $\varepsilon \to 0$. Posle uzimanja limesa $\varepsilon \to 0$, NE ZAVISE EKSPLICITNO OD μ, MADA ZAVISE IMPLICITNO OD μ JER λ i m^2 zavise od μ.

OVA ŠEMA "NE RADI" ZA BEZMASENE TEORIJE. $(m \to 0)$

② MOMENTUM SCHEMA

SUBTRAKCIONA TAČKA $p^2 = -M^2$

- $\Gamma^{(2)}(p^2=-M^2) = -M^2 - m^2$
- $\left.\frac{\partial \Gamma^{(2)}}{\partial p^2}\right|_{-M^2} = 1 \quad \Rightarrow \delta Z = 0$

$$\Rightarrow \delta m^2 = \frac{\lambda m^2}{32\pi^2}\left(\frac{2}{\varepsilon} + 1 - \gamma - \ln\frac{m^2}{4\pi\mu^2}\right)$$

- $\Gamma^{(4)}\Big|_{s=t=u=-\frac{4}{3}M^2} = -i\lambda\mu^{\varepsilon}$

$$\Rightarrow \delta\lambda = \frac{3\lambda^2}{32\pi^2}\left(\frac{2}{\varepsilon} - \gamma + \ln\frac{4\pi\mu^2}{m^2} - A\left(-\frac{4}{3}M^2, m^2\right)\right)$$

RENORMALIZOVANE 1PI Grinove fje su

$$\Gamma^{(2)} = p^2 - m^2$$

$$\Gamma^{(4)} = i\mu^{\varepsilon}\left\{-\lambda + \frac{\lambda^2}{32\pi^2}\left(A\left(-\frac{4M^2}{3}, m^2\right) - A(s,m^2) - A(u,m^2) - A(t,m^2)\right)\right\}$$

KAD $\varepsilon \to 0$ $\Gamma^{(4)}$ ne zavisi EKSPLICITNO OD μ

ALI ZAVISI OD μ PREKO m^2, λ.

③ MINIMALNA SUBTRAKCIJA (MS)

$\delta m^2, \delta\lambda$ se TAKO BIRAJU DA SE PONIŠTI SAMO DIV. DEO

$$\delta m^2 = \frac{\lambda m^2}{16\pi^2\varepsilon}$$

$$\delta\lambda = \frac{6\lambda^2}{32\pi^2\varepsilon}$$

$$\Gamma^{(2)} = p^2 - m^2 + \frac{\lambda^2 m^2}{32\pi^2}\left(1 - \gamma - \ln\frac{m^2}{4\pi\mu^2}\right)$$

$$\Gamma^{(4)} = -i\lambda\mu^{\varepsilon} + \frac{i\lambda^2\mu^{2\varepsilon}}{32\pi^2}\left(-3\gamma - 3\ln\frac{m^2}{4\pi\mu^2} - A(s,m^2) - A(u,m^2) - A(t,m^2)\right)$$

④ $\overline{MS}$ (MODIFIKOVANA MINIMALNA SUBTRAKCIJA

δm^2 i $\delta\lambda$ se TAKO BIRAJU DA SE PONIŠTI DIV. član

i γ i $\ln 4\pi$

$$\delta m^2 = \frac{\lambda^2 m^2}{32\pi^2}\left(\frac{2}{\varepsilon} - \gamma + \ln 4\pi\right)$$

$$\delta\lambda = \frac{\lambda^2}{32\pi^2}\left(\frac{6}{\varepsilon} - 3\gamma + 3\ln 4\pi\right)$$

RENOR. GRINOVE fje

$$\Gamma^{(2)} = p^2 - m^2 + \frac{\lambda^2 m^2}{32\pi^2}\left(1 - \ln\frac{m^2}{\mu^2}\right)$$

$$\Gamma^{(4)} = -i\lambda\mu^{\varepsilon} + \frac{i\lambda^2\mu^{2\varepsilon}}{32\pi^2}\left(-3\ln\frac{m^2}{\mu^2} - A(s, m^2) - A(u, m^2) - A(t, m^2)\right)$$

m^2 NIJE FIZIČKA MASA ČESTICE. FIZIČKA MASA m_{ph} JE ODREĐENA SA $\Gamma^{(2)}(p^2 = m_{ph}^2) = 0$

$$m_{ph}^2 = m^2 - \frac{\lambda^2 m^2}{32\pi^2}\left(1 - \ln\frac{m^2}{\mu^2}\right)$$

- MORA VAŽITI $\mu\frac{dm_{ph}}{d\mu} = 0$, JER FIZIKA NE SME DA ZAVISI OD μ (RGE)
- MASA m I KAPLING λ SU FUNKCIJE μ!

TEORIJA JE RENORMALIZABILNA AKO "DIVERGENCIJE" IMAJU ISTI OBLIK KAO ČLANOVI U POČETNOM LAGRANŽIJANU, TJ. AKO SE NE PROIZVODE NOVE DIVERGENCIJE. DRUGIM REČIMA $\mathcal{L}_{CT}$ IMA ISTI OBLIK KAO I $\mathcal{L}$. MEĐUTIM TO NIJE DOVOLJNO. MORA SE JOŠ POKAZATI DA RENORMALIZACIJOM KONSTANTI (MASE, λ, ..) I POLJA SE TE DIVERGENCIJE MOGU ABSORBOVATI

RENORMALIZACIONA GRUPA

RENORMALIZABILNA TEORIJA IMA proizvoljnost U IZBORU KINEMATIČKE TAČKE ZA DEFINISANJE MASA I KONSTANTI INTERAKCIJE.
Kontračlanovi moraju poništiti DIVERGENCIJE u 1PI
Različiti izbori SUPTRAKCIONE TAČKE SU RAZLIČITE REN.-ŠEME, A SVE ŠEME SU PODJEDNAKO DOBRE. Fizika NE SME DA ZAVISI OD IZBORA SUPTRAKCIONE TAČKE, FIZIČKI SADRŽAJ TEORIJE JE INVARIJANTAN NA PROMENU RŠ. TE TRANSFORMACIJE SU RG

$$S(g', m', \mu') = S(g, m, \mu) \quad (S-\text{MATRICA})$$

Gell-Mann-Low (1954) QED
Stueckelberg, Peterman ; Bogoljubov-Širkov (1959) Wilson (1969)
Callan-Symanzik (1970) 't Hooft Weinberg (MS)

$$\mathcal{L}_0 = \mathcal{L}_R + \Delta\mathcal{L}_R = \mathcal{L}_{R'} + \Delta\mathcal{L}_{R'}$$

$$\phi_0 = \sqrt{Z(R)}\,\phi_R = \sqrt{Z(R')}\,\phi_{R'} \Rightarrow \phi_{R'} = \underbrace{\sqrt{\frac{Z(R)}{Z(R')}}}_{\text{KONAČNO}}\,\phi_R$$

slično za mase i kaplinge!

IZBOR μ (UDR) ODNOSNO (SUPT. TAČKE M) JE SAMO MEĐUKORAK U RAČUNU.

Atomska fizika koja "radi" NA SKALI 10^{-8} cm je nezavisna od QFT na skali 10^{-13} cm

$$L_0 = \frac{1}{2}(\partial\phi_0)^2 - \frac{m_0^2}{2}\phi_0^2 - \frac{\lambda_0}{4!}\phi_0^4$$

$$L_0 = L + L_{CT}$$

$$L = \frac{1}{2}(\partial\phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\mu^\varepsilon\phi^4$$

$$L_{CT} = \frac{1}{2}\delta Z(\partial\phi)^2 - \frac{\delta m^2}{2}\phi^2 - \frac{\delta\lambda}{4!}\mu^\varepsilon\phi^4$$

- $G_0^{(n)} = \langle\Omega| T\phi_0(x_1)\ldots\phi(x_n)|\Omega\rangle = Z^{n/2}\langle\Omega|T\phi(x_1)\ldots\phi(x_n)|\Omega\rangle = Z^{n/2}G^{(n)}$

- $Z_0[J_0] = e^{iW_0[J_0]} = \int\mathcal{D}\phi_0\, e^{i\int\mathcal{L}_0(\phi_0) + i\int J_0\phi_0}$

$$= \int\mathcal{D}\phi\, e^{i\int\left[\frac{Z}{2}(\partial\phi)^2 - \frac{m^2+\delta m^2}{2}\phi^2 - \frac{(\lambda+\delta\lambda)\mu^\varepsilon\phi^4}{4!}\right] + i\int \overbrace{J_0\sqrt{Z}}^{J}\phi}$$

$$= Z[J] = e^{iW[J]}$$

$$\Gamma_0[\varphi_0] = W_0[J_0] - \int J_0\varphi_0 = W[J] - \int J\phi = \Gamma[\phi]$$

1PI su $\Gamma^{(n)}(x_1,\ldots,x_n) = \dfrac{\delta^n\Gamma[\varphi]}{\delta\varphi(x_1)\ldots\delta\varphi(x_n)}$

$$\Rightarrow \tilde{\Gamma}_0^{(n)}(p_1,\ldots p_n,\lambda_0,m_0,\varepsilon) = Z^{-n/2}\underbrace{\tilde{\Gamma}^{(n)}((p_1\ldots p_n),\lambda(\mu),m(\mu),\mu,\varepsilon)}_{\text{KONAČNO KAD } \varepsilon\to 0}$$

m_0, λ_0 divergiraju

m, λ su konačni i zavise od:

$m = m(m_0,\lambda_0,\mu,\varepsilon)$

$\lambda = \lambda(m_0,\lambda_0,\mu,\varepsilon)$

$Z = Z(\varepsilon,\lambda)$

Gole Greenove fje ne zavise od μ! Teorija je kompletno određena za $\mathcal{L}_0$; izmena μ je kompenzovana izmenama $m(\mu)$ i $\lambda(\mu)$

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$$\mu \frac{d\tilde{\Gamma}_0^{(n)}}{d\mu} = 0 \qquad \text{(držimo } m_0 \text{ i } \lambda_0 \text{ fiksnim)}$$

$$\left[\mu\frac{\partial}{\partial\mu} + \mu\frac{\partial\lambda}{\partial\mu}\frac{\partial}{\partial\lambda} + \mu\frac{\partial m}{\partial\mu}\frac{\partial}{\partial m} - \frac{n}{2}\mu\frac{\partial \ln Z}{\partial\mu}\right]\tilde{\Gamma}^{(n)}(p, m, \mu, \lambda, \varepsilon) = 0$$

$$\mu\frac{d\lambda}{d\mu} = \beta\left(\lambda, \frac{m}{\mu}, \varepsilon\right) \qquad \longrightarrow \quad \beta\text{-fja}$$

$$\frac{1}{2}\mu\frac{d\ln Z}{d\mu} = \gamma_\phi\left(\lambda, \frac{m}{\mu}, \varepsilon\right) \qquad \text{anom. dim. polja}$$

$$\mu\frac{\partial \ln m}{\partial\mu} = \gamma_m\left(\lambda, \frac{m}{\mu}, \varepsilon\right) \qquad \text{-- '' --} \quad \text{mase}$$

$$\boxed{\left[\mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial\lambda} + m\gamma_m\frac{\partial}{\partial m} - n\gamma_\phi\right]\tilde{\Gamma}^{(n)}(p_i, m, \mu, \lambda, \varepsilon) = 0} \quad \text{RGE}$$

u MS semi β-fja i γ_m, γ_ϕ ovise samo od λ a ne od $\frac{m}{\mu}$

$$\delta\lambda = \frac{3\lambda^2}{16\pi^2\varepsilon} + O(\lambda^2)$$

$$\delta Z = 1 + O(\lambda^2)$$

$$\delta m^2 = \frac{\lambda m^2}{(4\pi)^2\varepsilon}$$

$$\lambda_0 = \frac{\mu^\varepsilon(\lambda+\delta\lambda)}{Z^2} = \mu^\varepsilon\left(\lambda + \frac{3\lambda^2}{16\pi^2\varepsilon}\right)$$

$$0 = \mu\frac{d\lambda_0}{d\mu} = \varepsilon\mu^\varepsilon\left(\lambda + \frac{3\lambda^2}{16\pi^2\varepsilon}\right) + \mu^\varepsilon\left(1 + \frac{6\lambda}{16\pi^2\varepsilon}\right)\mu\frac{d\lambda}{d\mu}$$

$$\beta = -\frac{\varepsilon\left(\lambda + \frac{3\lambda^2}{16\pi^2\varepsilon}\right)}{1 + \frac{6\lambda}{16\pi^2\varepsilon}} = -\varepsilon\lambda + \frac{3\lambda^2}{16\pi^2} \longrightarrow \frac{3\lambda^2}{16\pi^2}$$

$$\mu\frac{d\lambda}{d\mu} = \frac{3\lambda^2}{16\pi^2} \Rightarrow \boxed{\lambda(\mu) = \lambda(\mu')\frac{1}{1 - \frac{3\lambda(\mu')}{16\pi^2}\ln\frac{\mu}{\mu'}}}$$

$$m_0^2 = m^2 + \frac{\lambda m^2}{16\pi^2\varepsilon}$$

$$0 = 2m\mu\frac{dm}{d\mu}\left(1+\frac{\lambda}{16\pi^2\varepsilon}\right) + \frac{m^2}{16\pi^2\varepsilon}\,\frac{-\varepsilon\left(\lambda+\frac{3\lambda^2}{16\pi^2\varepsilon}\right)}{1+\frac{6\lambda}{16\pi^2\varepsilon}}$$

$$2m\mu\frac{dm}{d\mu} = \frac{m^2}{16\pi^2\varepsilon}\,\frac{\varepsilon\left(\lambda+\frac{3\lambda^2}{16\pi^2\varepsilon}\right)}{\left(1+\cancel{\frac{\lambda}{16\pi^2\varepsilon}}\right)\left(1+\cancel{\frac{6\lambda}{16\pi^2\varepsilon}}\right)} = \frac{m^2\lambda}{16\pi^2}$$

$$\gamma_m = \mu\frac{1}{m}\frac{dm}{d\mu} = \underline{\underline{\frac{\lambda}{32\pi^2}}}$$

ŠTA SE DEŠAVA KADA SKALIRAMO IMPULSE? $p_i \to t\,p_i$

$$\Gamma[\varphi] = \sum \frac{1}{n!}\int \underbrace{d^dx_1 \dots d^dx_n}_{-nd}\;\underbrace{\varphi(x_1)\dots\varphi(x_n)}_{n\left(\frac{d}{2}-1\right)}\;\Gamma^{(n)}(x_1\dots x_n)$$

$$\left[\Gamma^{(n)}(x_1\dots x_n)\right] = n\left(\frac{d}{2}+1\right)$$

$$\tilde\Gamma^{(n)}(p_1\dots p_n)(2\pi)^d\underbrace{\delta^{(d)}(p_1+\dots+p_n)}_{-d} = \int \underbrace{dx_1\dots dx_n}_{-nd}\;\underbrace{\Gamma^{(n)}(x_1\dots x_n)}_{n\left(\frac{d}{2}+1\right)}\;e^{+i(p_1x_1+\dots p_nx_n)}$$

$$\Rightarrow \left[\tilde\Gamma^{(n)}(p_1\dots p_n)\right] = d\left(1-\frac{n}{2}\right)+n \equiv D$$ – <u>kanonska dimenzija</u>

$\tilde\Gamma^{(n)}$ je homogena fja.

$$\tilde\Gamma^{(n)}(tp_i, tm, t\mu, \lambda) = t^D\,\tilde\Gamma^{(n)}(p_i, m, \mu, \lambda)$$

$$t\frac{d}{dt}\tilde\Gamma^{(n)}(tp, tm, t\mu, \lambda) = D\,\tilde\Gamma^{(n)}(tp, tm, t\mu, \lambda)$$

$$t\frac{d}{dt}\tilde\Gamma^{(n)}(tp, tm, t\mu, \lambda) = \left(tp\frac{\partial}{\partial(tp)} + tm\frac{\partial}{\partial(tm)} + t\mu\frac{\partial}{\partial(t\mu)}\right)\cdot \tilde\Gamma^{(n)}(tp, tm, t\mu)$$

$$\underline{\left(tp_i\frac{\partial}{\partial(tp_i)} + tm\frac{\partial}{\partial(tm)} + t\mu\frac{\partial}{\partial t\mu}\right)\tilde\Gamma^{(n)}(tp_i, tm, t\mu) = D\,\tilde\Gamma(tp, tm, t\mu, \lambda)}$$

$\uparrow$ ovde se

... DO D

apsorbijeć $mt \to m$, $\mu t \to \mu$

$$\Rightarrow \boxed{\left(t\frac{\partial}{\partial t} + m\frac{\partial}{\partial m} + \mu\frac{\partial}{\partial \mu}\right)\tilde{\Gamma}^{(n)}(tp, m, \mu) = D\,\tilde{\Gamma}^{(n)}(tp_i, m, \mu, \lambda)}$$

ELIMINACIJOM $\mu\frac{\partial}{\partial\mu}$ $\Rightarrow$

$$\boxed{\left[-t\frac{\partial}{\partial t} + \beta\frac{\partial}{\partial\lambda} - n\gamma_\phi + m(\gamma_m - 1)\frac{\partial}{\partial m} + D\right]\tilde{\Gamma}^{(n)}(tp, \lambda, m, \mu) = 0} \quad (*)$$

Ova jednačina govori o tome kako $\tilde{\Gamma}^{(n)}$ reaguje na promenu skale.

IF $m = 0$ $\Rightarrow$ klasična teorija je scale invarijant.

Ovo nije tačno za kvantnu teoriju zbog $\leftarrow$

- U (MS šemi) β, γ_m, γ_ϕ ne zavise od m/μ, ~~t~~

Ovo odgovara bare bones

Rešavamo $(*)$ u MS

$$\tilde{\Gamma}^{(n)}(tp, m, \lambda, \mu) = f(t)\,\tilde{\Gamma}^{(n)}(p, \tilde{m}(t), \tilde{\lambda}(t), \mu)$$

Izmena impulsa $p_i \to tp_i$ kompenzovana je izmenom

$m \to \tilde{m}(t)$

$\lambda \to \tilde{\lambda}(t)$ $\rightarrow$ running c.c

Diferenciram po t

$$\frac{\partial}{\partial t}\tilde{\Gamma}^{(n)}(tp, m, \lambda, \mu) = \frac{df}{dt}\tilde{\Gamma}^{(n)}(p, \tilde{m}(t), \tilde{\lambda}(t), \mu) + f(t)\left(\frac{\partial\tilde{m}}{\partial t}\frac{\partial\tilde{\Gamma}^{n}}{\partial\tilde{m}} + \frac{\partial\tilde{\lambda}}{\partial t}\frac{\partial\tilde{\Gamma}^{(n)}}{\partial\tilde{\lambda}}\right)(p, \tilde{m}, \tilde{\lambda}, \mu)$$

$$t\frac{\partial}{\partial t}\tilde{\Gamma}^{(n)}(tp, m, \lambda, \mu) = \left[t\frac{df}{dt} + ft\frac{\partial\tilde{m}}{\partial t}\frac{\partial}{\partial\tilde{m}} + ft\frac{\partial\tilde{\lambda}}{\partial t}\frac{\partial}{\partial\tilde{\lambda}}\right)\frac{1}{f}\tilde{\Gamma}^{(n)}(tp, m, \lambda, \mu)$$

$$\Rightarrow\left[-t\frac{\partial}{\partial t} + \frac{1}{f}\frac{df}{dt} + t\frac{\partial\tilde{m}}{\partial t}\frac{\partial}{\partial m} + \frac{\partial\tilde{\lambda}}{\partial t}\frac{\partial}{\partial\lambda}\right]\tilde{\Gamma}^{(n)}(tp, m, \lambda, \mu) = 0$$

POREĐENJEM

$$t\frac{\partial\tilde\lambda(t)}{\partial t}=\beta(\tilde\lambda(t))$$

$$t\frac{\partial\tilde m(t)}{\partial t}=\tilde m\,(\gamma_m(\tilde\lambda)-1)$$

$$\frac{t}{f}\frac{df}{dt}=D-n\gamma_\phi(\tilde\lambda)$$

$$\tilde\lambda(t=1)=\lambda$$

$$\tilde m(t=1)=m$$

$$\Rightarrow f=t^D e^{-n\int_1^t \frac{\gamma_\phi(\tilde\lambda(s))}{s}ds}$$

Dakle:

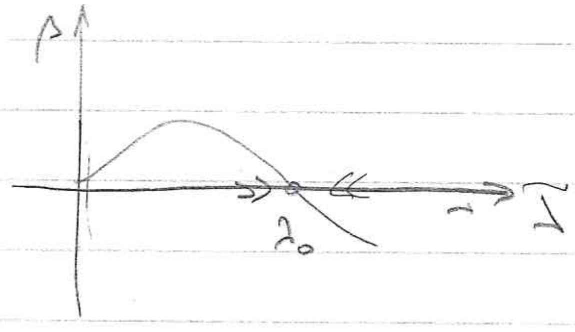
$$\Gamma^{(n)}(tp,m,\lambda,\mu)=t^{4-n}\,e^{-n\int_1^t\frac{\gamma_\phi(\tilde\lambda(s))}{s}ds}\,\tilde\Gamma^{(n)}(p,\tilde m(t),\tilde\lambda(t),\mu)$$

Fizika na velikim energijama određena je sa $\tilde m(t), \tilde\lambda(t)$

Na velikim energijama teorija koja je scale invarijantna

- nule β fje imaju specijalnu ulogu

$\beta(\lambda_0)=0$ λ_0 — FIKSNA TAČKA

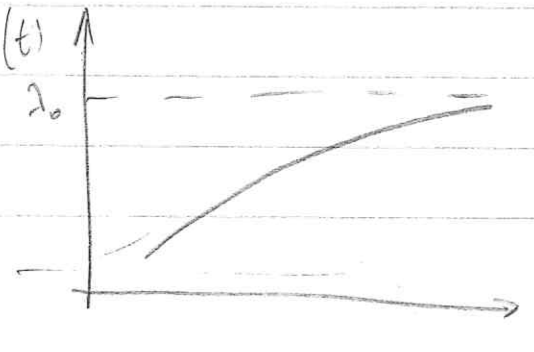


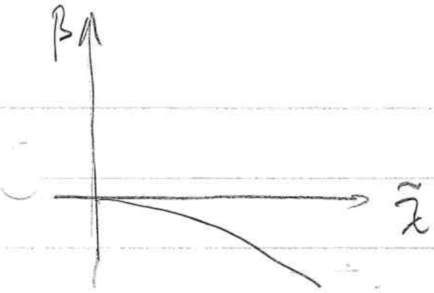
$$\beta(\tilde\lambda)=t\frac{\partial\tilde\lambda}{\partial t}$$

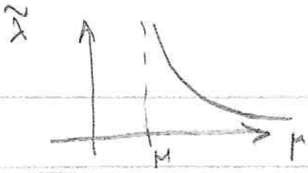
- $\lambda>\lambda_0$; $\beta<0\Rightarrow\frac{\partial\tilde\lambda}{\partial t}<0\Rightarrow$ t raste λ opada do λ_0
- $\lambda<\lambda_0$; $\beta>0\Rightarrow\frac{\partial\tilde\lambda}{\partial t}>0\Rightarrow$ t raste λ raste do λ_0

Dakle kod $t\to\infty$ $\tilde\lambda(t)\to\lambda_0$ (UV stab. F.T.)

$$\beta(\lambda)\approx\beta'(\lambda_0)(\lambda-\lambda_0)+\dots$$



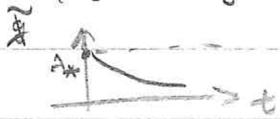


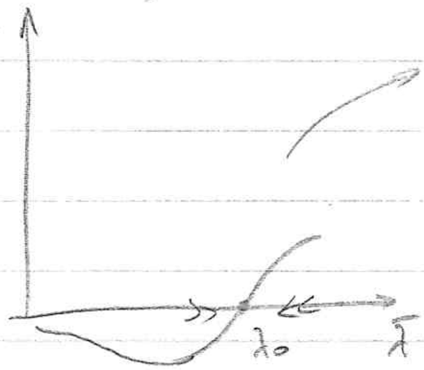


$\lambda_0 = 0$

$\frac{d\tilde{\lambda}}{dt} < 0 \Rightarrow \boxed{\tilde{\lambda}(t \to \infty) = 0}$ AS. SLOBODA

$\lambda_0 = 0$ je UV FIKSNA TAČKA

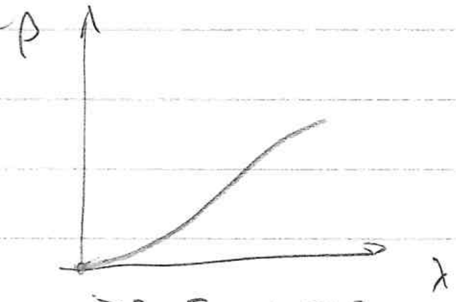




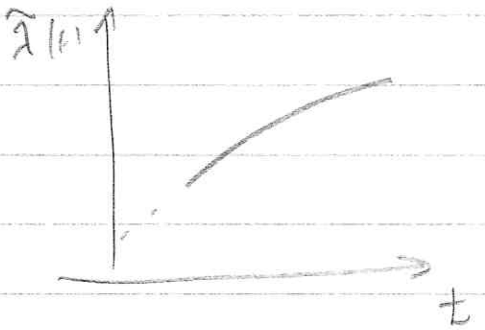
$\tilde{\lambda} > \lambda_0$; $\beta > 0$ t opada ; $\tilde{\lambda}$ opada do λ_0

$\lambda < \lambda_0$ $\beta < 0$ t opada, $\tilde{\lambda}$ raste do λ

$\tilde{\lambda}(t=0) = \lambda_0$ IC FIKSNA TAČKA



$\Longleftarrow$



IR FIKSNA TAČKA

$\boxed{\varphi^4}$ $\beta(\lambda) = \frac{3\lambda^2}{16\pi^2} > 0$ SA PORASTOM IMPULSA EФ K.K. raste (PERTURBATIVNU TEORIJU MOŽEMO DA PRIMENJUJEMO na malim energijama)

$$\beta(\tilde{\lambda}(t)) = t\frac{d\tilde{\lambda}}{dt} = 3\frac{\tilde{\lambda}^2(t)}{16\pi^2}$$

$$\boxed{\tilde{\lambda}(t) = \frac{\lambda(0)}{1 - \frac{3}{16\pi^2}\lambda(0)\ln t}}$$

za $\ln t_0 = \frac{16\pi^2}{3\lambda(0)}$ $\tilde{\lambda} \to \infty$, Landauov singularitet

$P = P(m, \lambda, \mu) \rightarrow$ физичка ~~величина~~ (S, m_{ph})

$$\underbrace{\left(\mu \frac{\partial}{\partial \mu} + \beta \frac{\partial}{\partial \lambda} + m \gamma_m(\lambda) \frac{\partial}{\partial m}\right)}_{\mathcal{D}} P = 0$$

$$(\mathcal{D} + n\gamma_\phi)\, G^{(n)}(x_1 \ldots x_n, m, \lambda, \mu) = 0$$

$$(\mathcal{D} + 2\gamma_\phi)\, i\tilde{\Delta}(p^2) = 0$$

$$(\mathcal{D} + 2\gamma_\phi)\left(\frac{iZ}{p^2 - m_{ph}^2} + \text{OST.}\right) = 0$$

$$Z \frac{\overbrace{\mathcal{D} m_{ph}^2}^{=0}}{(p^2 - m_{ph}^2)^2} + (\mathcal{D} Z)\frac{1}{p^2 - m_{ph}^2} + \frac{2\gamma_\phi Z}{p^2 - m_{ph}^2} = 0$$

$$\boxed{\mathcal{D} Z + 2\gamma_\phi Z = 0}$$

$$S = \frac{1}{(\sqrt{Z})^{n+m}} \prod_i (p_i^2 - m_{ph}^2)\, \tilde{G}^{(n+m)}(p_i)$$

$$\mathcal{D} S = -\frac{n+m}{2} Z^{-\frac{(n+m)}{2} - 1} (\mathcal{D} Z)\, \tilde{G}^{(n+m)}(p_i) \prod_i (p_i^2 - m_{ph}^2)$$

$$+ \frac{1}{(\sqrt{Z})^{n+m}} \prod_i (p_i^2 - m_{ph}^2)\, \underbrace{\mathcal{D}\tilde{G}^{(n+m)}(p)}_{-(n+m)\gamma_\phi G^{(n+m)}}$$

$$= \left[+ \frac{n+m}{2} \frac{2\gamma_\phi}{(\sqrt{Z})^{n+m}} - \frac{(n+m)\gamma_\phi}{(\sqrt{Z})^{n+m}} \right] \prod_i (p_i^2 - m_{ph}^2)\, \tilde{G}^{(n+m)}$$

$$= 0 \quad \checkmark$$

NAPOMENE: – β ne zavisi od šeme

– $\beta = \underset{\downarrow}{b_0} g^3$

univ

$\boxed{\text{QED}}$

$$Z_1 = Z_2 = 1 - \frac{e^2}{8\pi^2\varepsilon}$$

$$Z_3 = 1 - \frac{e^2}{6\pi^2\varepsilon}$$

- $$e_0 = \mu^{\varepsilon/2} \frac{e}{\sqrt{Z_3}} = \mu^{\varepsilon/2} e \left(1 + \frac{e^2}{12\pi^2\varepsilon}\right)$$

$$0 = \frac{\varepsilon}{2} \mu^{\frac{\varepsilon}{2}-1} e \left(1 + \frac{e^2}{12\pi^2\varepsilon}\right) + \mu^{\varepsilon/2}\left(1 + \frac{e^2}{4\pi^2\varepsilon}\right)\frac{\partial e}{\partial\mu}$$

$$\beta = -\frac{\varepsilon}{2} e \left(1 - \frac{e^2}{6\pi^2\varepsilon}\right) \xrightarrow[\varepsilon\to 0]{} \underline{\underline{\frac{e^2}{12\pi^2}}}$$

$$m_0 = m\left(1 - \frac{3e^2}{8\pi^2\varepsilon}\right) \Rightarrow \gamma_m = -\frac{3e^2}{8\pi^2}$$

- $$\gamma_\psi = \frac{1}{2}\mu \frac{d \ln Z_\psi}{d\mu} = \frac{e^2}{16\pi^2}$$

- $$\gamma_A = \frac{e^2}{12\pi^2}$$

$$\boxed{e^2(\mu) = \frac{e^2(\mu')}{1 - \frac{e^2(\mu')}{6\pi^2}\ln\frac{\mu}{\mu'}}}$$ ili

$$\alpha(\mu) = -\frac{3\pi}{2}\frac{1}{\ln\frac{\mu}{\Lambda_{QED}}}$$

$$\alpha(m_e = 0{,}51\,\text{MeV}) = \frac{1}{137} \Rightarrow \Lambda_{QED} = 10^{286}\,\text{eV}$$

(Landauov pol)

$\frac{1}{137}$ smo trampili (zamenili) sa Λ_{QED}

(DIM. TRANSMUTACIJA)

$$\alpha(E \sim 90\,\text{GeV}) = \frac{1}{127}$$