$\mathcal{M} = \varepsilon^*_\alpha(k_1)\cdots \quad \mathcal{M}^{\alpha\beta\cdots}(k_1,\dots)$

$A^\mu \sim \varepsilon^\mu_r(k)\; e^{\mp ikx}$

$A^\mu \to A^\mu + \partial^\mu \Lambda(x) \quad + \; \Lambda(x) = \tilde{\Lambda}(k)\, e^{\pm ikx} \Longrightarrow$

$$\underline{\varepsilon^\mu_r(\vec{k}) \;\to\; \varepsilon^\mu_r(\vec{k}) + i k^\mu \tilde{\Lambda}(k)}$$

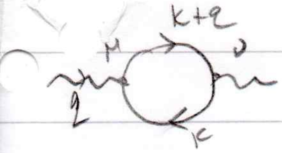
$\Longrightarrow \quad \boxed{k_{1\alpha}\cdots \; \mathcal{M}^{\alpha\cdots}(k_1,\dots) = 0}$

gauge invarijantnost:

$k_\mu \Gamma^\mu(p,k,p') = S_F^{-1}(p+k) - S_F^{-1}(k)$ nije nula

ali jeste on-shell !!

Polarizacija vakuuma



$$i\Pi_2^{\mu\nu}(q) = i^2(-ie)^2(-)\int \frac{d^4k}{(2\pi)^4}\frac{\mathrm{Tr}[\gamma^\mu(\not k+m)\gamma^\nu(\not k+\not q+m)]}{[k^2-m^2+i\varepsilon][(k+q)^2-m^2+i\varepsilon]}$$

1PI $\equiv i\Pi^{\mu\nu}(q) =$ + + ...

Word-ov id. $\quad q_\mu \Pi^{\mu\nu}(q) = 0 \quad \Longrightarrow$

$$\boxed{\Pi^{\mu\nu}(q) = (q^2 g^{\mu\nu} - q^\mu q^\nu)\,\Pi(q^2)}$$

$\Pi^{\mu\nu}(q)$ нема пол у $q^2=0$; $\quad \Pi(q^2)$ регуларно у $q^2=0$

$\int d^4x\, e^{+iqx}\langle\Omega|T A_\mu(x) A_\nu(y)|\Omega\rangle = {}^{\mu}\,{}^{\nu} + {}^{\mu}\,{}^{\rho}$ 1PI ${}^{\sigma}\,{}^{\nu}$ + 1PI 1PI + ...

$$= \frac{-i}{q^2+i\varepsilon}\Big[\underbrace{g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}}_{\Pi_{T\mu\nu}} + \xi\underbrace{\frac{q_\mu q_\nu}{q^2}}_{\Pi_{L\mu\nu}}\Big] + \frac{-i}{q^2+i\varepsilon}\big[\Pi_{T\mu\rho} + \cancel{\xi\,\Pi_{L\mu\rho}}\big]\cdot i\,\Pi_T^{\rho\sigma} q^2\,\Pi(q^2)$$

$$\cdot\frac{-i}{q^2+i\varepsilon}\big[\Pi_{T\sigma\nu} + \cancel{\xi\,\Pi_{L\sigma\nu}}\big] + \dots$$

$$= \frac{-i}{q^2+i\varepsilon}\Big(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\Big)\big(1 + \Pi(q^2) + \Pi^2(q^2) + \dots\big) - \frac{i}{q^2+i\varepsilon}\,\frac{\xi q_\mu q_\nu}{q^2}$$

$$= \boxed{\frac{-i}{q^2(1-\Pi(q^2))+i\varepsilon}\Big(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\Big) - \frac{i}{q^2}\,\frac{\xi q_\mu q_\nu}{q^2}}$$

Deo $q^\mu q^\nu$ ne daje doprinos S-matrici; Mogu i $\xi=0$

FOTON TREBA DA OSTANE BEZMASEN U SVIM RED. TP

$$q^2\Pi(q^2)\big|_{q^2=0}=0$$

$$q^2\Pi(q^2) = q^2\left(\Pi(0)+\hat{\Pi}(q^2)\right)$$

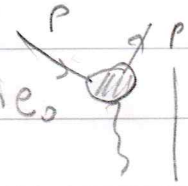
$$\frac{-iP^T_{\mu\nu}}{q^2+i\varepsilon} \to \frac{-iP^T_{\mu\nu}}{q^2\left(1-\Pi(0)-\hat{\Pi}(q^2)\right)}$$

$$= \frac{-iP^T_{\mu\nu}}{q^2\left(1-\Pi(0)\right)\left(1-\frac{\hat{\Pi}(q^2)}{1-\Pi(0)}\right)}$$

$$= \frac{-iP^T_{\mu\nu}Z_3}{q^2\left(1-\frac{\hat{\Pi}(q^2)}{1-\Pi(0)}\right)}$$

$\boxed{Z_3 = \frac{1}{1-\Pi(0)}}$ renormalizacione konstante FOTONA

o renormalizacija naelektrisanja.



$$-ie_0 \Big|_{q=0} = -ie_0\sqrt{Z_3}\,Z_2\,\Gamma^\mu(p',p)\Big|_{q=0}$$

$$= \sqrt{Z_3}\,Z_2\,\frac{-ie_0\gamma^\mu}{Z_1} = -ie_0\,\cancel{\frac{Z_2}{Z_1}}\sqrt{Z_3}\,\gamma^\mu$$

$$= -ie_0\sqrt{Z_3}\,\gamma^\mu$$

$\Rightarrow \boxed{e = \sqrt{Z_3}\,e_0}$ bare

phys. charge

e_0 – golo naelektrisanje
e – fizičko naelektrisanje

IZRAČUNAVANJE $\Pi_2(q)$

$$i\Pi_2^{\mu\nu}(q) = -e_0^2 \int \frac{d^4k}{(2\pi)^4} \frac{Tr\left[\gamma^\mu(\not k + m_0)\gamma^\nu(\not k + \not q + m_0)\right]}{(k^2 - m_0^2 + i\varepsilon)((k+q)^2 - m_0^2 + i\varepsilon)}$$

$e_0 = e + O(\alpha^2)$
$m_0 = m + O(\alpha^2)$ $\Rightarrow$ u 2 RT.P. $m_0 \to m$, $e_0 \to e$

$$i\Pi_2^{\mu\nu}(q) = -4e^2 \int_0^1 dx \int \frac{d^4k}{(2\pi)^4} \frac{k^\mu(k+q)^\nu + k^\nu(k+q)^\mu - g^{\mu\nu}(k\cdot(k+q) - m^2)}{(k^2 - m^2 + x(2kq + q^2) + i\varepsilon)^2}$$

SMENA $\quad l^\mu = k^\mu + xq^\mu$

$$= -4e^2 \int_0^1 dx \int \frac{d^4l}{(2\pi)^4} \frac{2l^\mu l^\nu + 2x^2q^\mu q^\nu - 2xq^\mu q^\nu - g^{\mu\nu}(l^2 + x^2q^2 - xq^2 - m^2) + l.}{(l^2 - \Delta + i\varepsilon)^2}$$

$$\Delta = m^2 - q^2x + q^2x^2$$

$$\xrightarrow[\text{Wick}]{} -4ie^2 \int_0^1 dx \int \frac{d^4l_E}{(2\pi)^4} \frac{-\frac{1}{2}g^{\mu\nu}l_E^2 + g^{\mu\nu}l_E^2 - 2x(1-x)q^\mu q^\nu - g^{\mu\nu}(x^2q^2 - xq^2 - m^2)}{(l_E^2 + \Delta)^2}$$

① cut-off $\quad 0 \leq |l_E| \leq \Lambda$

$$i\Pi^{\mu\nu} \sim \int_0^\Lambda l_E^3\, dl_E \frac{[l_E^2 g^{\mu\nu}, 1]}{l_E^4} \sim g^{\mu\nu}\Lambda^2$$

Nedostaje $q^\mu q^\nu$ deo — narušava Vordov. identitet
foton dobija masu i to beskonačnu
cut ~~off~~ narušava gauge simetriju

② Pauli - Vilors Reg.
$\to$ više fermionskih polja $\quad$ I·

DIMENZIONA REGULARIZACIJA

't Hooft Veltman NPB 44, 189 (1972)

$D = 4 - \varepsilon$

$$\int \frac{d^4 l}{(2\pi)^4} \frac{1}{(l^2 - \Delta)^n} = (-1)^n i \int \frac{d^4 l_E}{(2\pi)^4} \frac{1}{(l_E^2 + \Delta)^n}$$

$$\longrightarrow i(-1)^n \int \frac{d^D l_E}{(2\pi)^D} \frac{1}{(l_E^2 + \Delta)^n}$$

$$= \frac{i(-1)^n}{(2\pi)^D} \int d\Omega_D \int_0^\infty dl_E \frac{l_E^{D-1}}{(l_E^2 + \Delta)^n}$$

- $\int d\Omega_D = \frac{2\pi^{D/2}}{\Gamma\left(\frac{D}{2}\right)}$ (POVRŠINA jedinične D-dim. sfere)
- $\int_0^\infty dl_E \, l_E^{D-1} (l_E^2 + \Delta)^{-n} = \frac{1}{2} \int_0^\infty dl_E^2 (l_E^2)^{\frac{D}{2}-1} (l_E^2 + \Delta)^{-n} = \left(\text{SMENA } x = \frac{\Delta}{l_E^2 + \Delta}\right)$

$$= \frac{1}{2} \Delta^{\frac{D}{2} - n} \int_0^1 dx \, x^{n - \frac{D}{2} - 1} (1 - x)^{\frac{D}{2} - 1}$$

$$= \frac{1}{2} \frac{1}{\Delta^{n - \frac{D}{2}}} \frac{\Gamma\left(n - \frac{D}{2}\right) \Gamma\left(\frac{D}{2}\right)}{\Gamma(n)}$$

$$\Rightarrow \int d^D l \frac{1}{(l^2 - \Delta)^n} = i(-1)^n \frac{2\pi^{\frac{D}{2}}}{\Gamma\left(\frac{D}{2}\right)} \frac{1}{2} \frac{1}{\Delta^{n - \frac{D}{2}}} \frac{\Gamma\left(n - \frac{D}{2}\right) \Gamma\left(\frac{D}{2}\right)}{\Gamma(n)}$$

$$= (-1)^n \frac{i \pi^{D/2}}{\Delta^{n - \frac{D}{2}}} \frac{\Gamma\left(n - \frac{D}{2}\right)}{\Gamma(n)}$$

$z \mapsto \Gamma(z)$ ima polove u $z = 0, -1, -2, \ldots$

$$\Gamma(-n + \varepsilon) = \frac{(-1)^n}{n!} \left(\frac{1}{\varepsilon} + \psi_1(n+1) + \mathcal{O}(\varepsilon) \right)$$

$$\psi_1(z) = \frac{d \ln \Gamma}{dz}$$

$$\psi_1(n+1) = 1 + \frac{1}{2} + \ldots + \frac{1}{n} - \gamma \qquad \gamma = 0{,}557\ldots$$

$$\Gamma(\varepsilon) = \frac{1}{\varepsilon} - \gamma$$

NEKOLIKO MODIFIKACIJA

1) $e\int d^4x\, \bar\Psi A\!\!\!/ \Psi \longrightarrow e\mu^{2-\frac{D}{2}}\int d^Dx\, \bar\Psi A\!\!\!/ \Psi$

$\mu \rightarrow$ MASENI PARAMETAR

$[\Psi] = \frac{D-1}{2}$

$[A] = \frac{D}{2}-1$

$[e] = 0$

$[d^Dx] = -D$

$[\mu^x \bar\Psi A\!\!\!/ \Psi] = D \Rightarrow \underline{x = 2-\frac{D}{2}}$

$$\boxed{e \rightarrow e\mu^{\frac{\varepsilon}{2}}}$$

2) $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$

$\gamma^\mu\gamma_\mu = D$

$\gamma^\mu\gamma_\nu\gamma_\mu = (2-D)\gamma_\nu, \ldots$

$Tr(I) = f(D)$; $f(4) = 4$

$Tr(\gamma_\mu\gamma_\nu) = f(D)\, g_{\mu\nu}$

$Tr(\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma) = f(D)\left(g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\nu\sigma} + g_{\mu\sigma}g_{\nu\rho}\right)$

Problem γ_5 u $D = 4-\varepsilon$!

- $$\int \frac{d^Dl}{(2\pi)^D} \frac{l^\mu l^\nu}{(l^2-\Delta)^n} = \frac{i(-1)^n}{2^D \pi^{D/2}} \left(-\frac{1}{2}\right) g^{\mu\nu} \frac{\Gamma(n-\frac{D}{2}-1)}{\Delta^{n-\frac{D}{2}-1}}$$

$$i\Pi_2^{\mu\nu}(q) = -4e^2\mu^\varepsilon \int \frac{d^Dl}{(2\pi)^D} \int_0^1 dx \, \frac{2l^\mu l^\nu - l^2 g^{\mu\nu} + q^\mu q^\nu (2x^2-2x) + g^{\mu\nu}(m^2 + xq^2 - x^2q^2)}{(l^2-\Delta)^2}$$

$$= \frac{-4e^2\mu^\varepsilon}{(2\pi)^D} \int_0^1 dx \left\{ 2\,\frac{-i\pi^{D/2}}{2} g^{\mu\nu} \frac{\Gamma(1-\frac{D}{2})}{\Delta^{1-\frac{D}{2}}} - \frac{(-i)\pi^{D/2}}{2} D g^{\mu\nu} \frac{\Gamma(1-\frac{D}{2})}{\Delta^{1-\frac{D}{2}}} \right.$$

$$+ \left(2x(x-1) q^\mu q^\nu + g^{\mu\nu}(m^2 + q^2 x(1-x)\right) \frac{i\pi^{D/2}}{\Delta^{2-\frac{D}{2}}} \Gamma\left(2-\frac{D}{2}\right)$$

$$\Gamma\left(2-\frac{D}{2}\right) = \Gamma\left(\frac{\varepsilon}{2}\right) = \frac{2}{\varepsilon} - \gamma + \sigma(\varepsilon)$$

$$\Gamma\left(1-\frac{D}{2}\right) = \Gamma\left(\frac{\varepsilon}{2}-1\right) = -\left(\frac{2}{\varepsilon} + 1 - \gamma + \sigma(\varepsilon)\right)$$

$$i\Pi_2^{\mu\nu} = -\frac{4e^2\mu^\varepsilon}{(2\pi)^D} \int_0^1 dx \left\{ -\frac{i\pi^{D/2} g^{\mu\nu}}{\Delta^{1-\frac{D}{2}}} \overbrace{\left(1-\frac{D}{2}\right)\Gamma\left(1-\frac{D}{2}\right)}^{\Gamma(2-\frac{D}{2})} + \right.$$

$$\left. + \left(2x(x-1) q^\mu q^\nu + g^{\mu\nu}(m^2 + q^2 x(1-x)\right) \frac{i\pi^{D/2}}{\Delta^{2-\frac{D}{2}}} \Gamma\left(2-\frac{D}{2}\right) \right\}$$

$$= \frac{-4e^2\mu^\varepsilon}{2^D\pi^D} i\pi^{D/2} \left(\frac{2}{\varepsilon} - \gamma + \ldots\right) \int_0^1 dx \left\{ \frac{-g^{\mu\nu}}{\Delta^{1-\frac{D}{2}}} + \left(2x(x-1) q^\mu q^\nu \right.\right.$$

$$\left. + g^{\mu\nu}(m^2 + q^2 x(1-x)) \frac{1}{\Delta^{2-\frac{D}{2}}} \right\}$$

$$= -\frac{4ie^2(\mu^2)^{\frac{\varepsilon}{2}}}{16\pi^2(4\pi)^{-\frac{\varepsilon}{2}}} \left(\frac{2}{\varepsilon} - \gamma + \sigma(\varepsilon)\right) \int_0^1 dx \left\{ -g^{\mu\nu}(\cancel{m^2} - q^2x + q^2x^2) \right.$$

$$\left. + g^{\mu\nu}(\cancel{m^2} + q^2(x-x^2)) + 2x(x-1) q^\mu q^\nu \right\} (m^2 - q^2x + q^2x^2)^{-\frac{\varepsilon}{2}}$$

$$\mu^\varepsilon (4\pi)^{\frac{\varepsilon}{2}} = 1 + \frac{\varepsilon}{2} \ln 4\pi\mu^2 + \ldots$$

$$\Delta^{-\frac{\varepsilon}{2}} = 1 - \frac{\varepsilon}{2} \ln \Delta + \ldots$$

$$i\Pi_2^{\mu\nu} = -\frac{ie^2}{4\pi^2}\left(\frac{2}{\varepsilon}-\gamma+O(\varepsilon)\right)\left(q^2g^{\mu\nu}-q^\mu q^\nu\right)\cdot 2\int_0^1 dx\, x(1-x)\left(1-\frac{\varepsilon}{2}\ln\frac{\Delta}{4\pi\mu^2}\right)$$

$$= -\left(q^2g^{\mu\nu}-q^\mu q^\nu\right)\frac{ie^2}{2\pi^2}\left\{\frac{1}{3\varepsilon}-\frac{\gamma}{6}-\int_0^1 dx\, x(1-x)\ln\frac{m_0^2-x(1-x)q^2}{4\pi\mu^2}\right.$$

- KOREKCIJA je TRANSVERZALNA

- $Z_3 = \frac{1}{1-\Pi_2(0)} \cong 1+\Pi_2(0) = -\frac{e_0^2}{6\pi^2\varepsilon} \approx \frac{-e^2}{6\pi^2\varepsilon}$

$$A_{\mu 0} = \sqrt{Z_3}\,A_\mu$$

$e = e_0\sqrt{Z_3}$ → RENORMALIZIRANO NAB.

za S-MATRICO člen $q^\mu q^\nu$ ne da doprinos zbog z.o. nab.

 $= -\frac{ig^{\mu\nu}e_0^2}{q^2\left(1-\Pi(q^2)\right)} \underset{O(\alpha)}{=} \frac{-ig^{\mu\nu}e_0^2}{q^2\left(1-\Pi_2(0)-\hat{\Pi}_2(q^2)\right)}$

$$= \frac{-ig^{\mu\nu}e_0^2}{q^2\left(1-\Pi_2(0)\right)\left(1-\frac{\hat{\Pi}_2(q^2)}{1-\Pi_2(0)}\right)} \approx -\frac{ig^{\mu\nu}e^2}{q^2\left(1-\hat{\Pi}_2(q^2)\right)} = -\frac{-ig^{\mu\nu}e^2}{q^2\left(1-\Pi_2(q^2)-\Pi_2(0)\right)}$$

$$\hat{\Pi}(q^2) \equiv \Pi_2(q^2)-\Pi_2(0)$$

$$\alpha_0 = \frac{e_0^2}{4\pi}$$

$$\alpha_0 \to \alpha_{eff}(q^2) = \frac{\alpha_0}{1-\Pi(q^2)} = \frac{\alpha}{1-\hat{\Pi}(q^2)} + O(\alpha)$$

$$\hat{\Pi}_2(q^2) = \Pi_2(q^2) - \Pi(0) = -\frac{2\alpha}{\pi}\int_0^1 dx\, x(1-x)\ln\frac{m^2}{m^2 - x(1-x)q^2 - i\varepsilon}$$

ANALITIČKA STRUKTURA IZRAZA $\hat{\Pi}_2(q^2)$

1) $q^2 < 0 \Rightarrow \hat{\Pi}_2(q^2) \in \mathbb{R}$

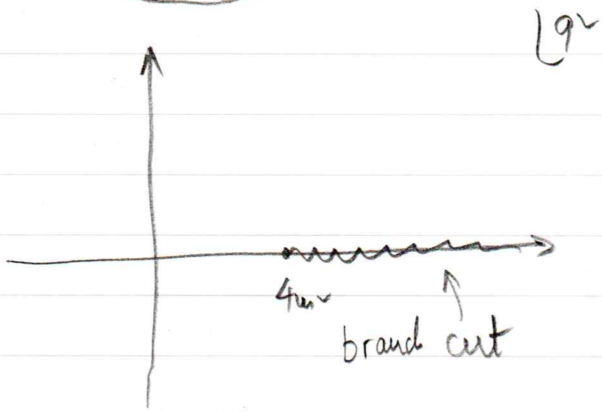
2) $q^2 > 0$ $z \mapsto \ln z$ ima branch cut

$$m^2 - x(1-x)q^2 = 0$$

$$q^2 = \frac{m^2}{x(1-x)}$$

$$\frac{\partial q^2}{\partial x} = \frac{m^2}{x^2(1-x)^2}(2x-1) = 0 \Rightarrow \underline{x = \frac{1}{2}}$$

$\Rightarrow q^2 = 4m^2$ prag za kreiranje e^-e^+ para



Note: $\frac{m^2}{q^2} - x + x^2 = 0$

$$x_{1,2} = \frac{1 \pm \sqrt{1 - \frac{4m^2}{q^2}}}{2} = \frac{1}{2} \pm \frac{\beta}{2}$$

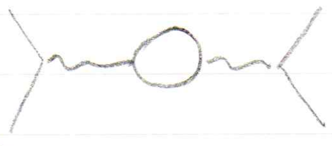
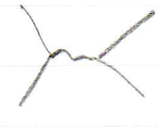
$$\mathrm{Im}\, \ln(-x + i\varepsilon) = \pm\pi \quad \text{za } x > 0$$

$$F \ln(-x \pm i\varepsilon) = \ln x - \ln(-1 \pm i\varepsilon)$$
$$= \ln x \pm i\pi$$

$$\mathrm{Im}\, \hat{\Pi}_2(q^2 \pm i\varepsilon) = \frac{2\alpha}{\pi} \int_0^1 dx\, (1-x)x\ \mathrm{Im}\ln\left(m^2 - x(1-x)q^2 \mp i x(1-x)\varepsilon\right)$$
$$= \frac{2\alpha}{\pi} \int_{\frac{1}{2}-\frac{\beta}{2}}^{\frac{1}{2}+\frac{\beta}{2}} dx\, x(1-x)\,(\mp\pi)$$
$$= \mp \frac{\alpha}{3} \sqrt{1 - \frac{4m^2}{q^2}} \left(1 + \frac{2m^2}{q^2}\right)$$

$$\mathrm{Disc}\, \hat{\Pi}(p^2) = 2i\, \mathrm{Im}\, \hat{\Pi}(p^2)$$

U SKLADU SA OPTIČKOM TEOREMOM

$2\,\mathrm{Im}\Big($  $\Big) = \int d\Pi\, \Big|$  $\Big|^2$