OPTIČKI TEOREM

S – MATRICA $\quad S = \mathbb{1} + iT$

"T" MATRICA

S MATRICA je UNITARNA

$$SS^+ = \mathbb{1} \Rightarrow (1+iT)(1-iT^+) = 1 \Rightarrow$$

$$\boxed{-i(T - T^+) = T^+T}$$

DVOČESTIČNO STANJE $|\vec{k}_1, \vec{k}_2\rangle \longrightarrow$ DVOČ. STANJE $|\vec{p}_1, \vec{p}_2\rangle$

$$\langle \vec{p}_1 \vec{p}_2 | T^+T | \vec{k}_1 \vec{k}_2\rangle = \sum_n \left(\prod_{i=1}^n \frac{d^3 q_i}{(2\pi)^3} \frac{1}{2E_i}\right) \langle \vec{p}_1, \vec{p}_2 | T^+ | \{\vec{q}_i\}\rangle\langle\{\vec{q}_i\}| T | \vec{k}_1, \vec{k}_2\rangle$$

intermediate states (kompletan skup)

$$iT = (2\pi)^4 \delta^{(4)}(p_1 + p_2 - k_1 - k_2)\, i\mathcal{M}$$

$$L.S = -i\left[\langle \vec{p}_1 \vec{p}_2 | T | \vec{k}_1, \vec{k}_2\rangle - \langle \vec{p}_1, \vec{p}_2 | T^+ | \vec{k}_1, \vec{k}_2\rangle\right] =$$

$$= -i(2\pi)^4 \delta^{(4)}(p_1 + p_2 - k_1 - k_2)\left[\mathcal{M}(k_1 k_2 \to p_1 p_2) - \mathcal{M}^*(\vec{p}_1, \vec{p}_2 \to \vec{k}_1, \vec{k}_2)\right]$$

$$D.S = \sum_n \left(\prod_{i=1}^n \frac{d^3 q_i}{(2\pi)^3} \frac{1}{2E_i}\right)(2\pi)^{4\cdot 2} \delta^{(4)}(p_1 + p_2 - k_1 - k_2)\, \mathcal{M}(\vec{k}_1, \vec{k}_2 \to \{\vec{q}_i\}) \cdot$$

$$\cdot \mathcal{M}^*(\vec{p}_1 \vec{p}_2 \to \{\vec{q}_i\})\, \delta^{(4)}\left(k_1 + k_2 - \sum_1^n q_i\right)$$

$\Longrightarrow$

$$-i\left(\mathcal{M}(k_1 k_2 \to p_1 p_2) - \mathcal{M}^*(p_1 p_2 \to k_1 k_2)\right) =$$

$$= \sum_n \left(\prod_{i=1}^n \frac{d^3 q_i}{(2\pi)^3} \frac{1}{2E_i}\right)(2\pi)^4 \delta^{(4)}\left(k_1 + k_2 - \sum q_i\right) \cdot \mathcal{M}(k_1 k_2 \to \{\vec{q}_i\})\, \mathcal{M}^*(p_1 p_2 \to \{q_i\})$$

$$\int d\Pi_n = \prod_{i=1}^n \int \frac{d^3 q_i}{(2\pi)^3} \frac{1}{2E_i} (2\pi)^4 \delta^{(4)}\left(p_1 + p_2 - \sum_1^n q_i\right)$$

integral po finalnim stanjima

(ZA BEZSPINSKE čestice $\mathcal{M}(k_1 k_2 \to p_1 p_2) = \mathcal{M}(p_1 p_2 \to k_1 k_2)$)

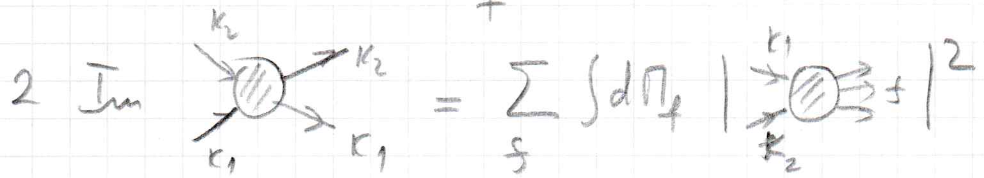
$$\boxed{-i\left[\mathcal{M}(k_1k_2\to p_1p_2)-\mathcal{M}^*(p_1p_2\to k_1k_2)\right]=\sum_f\int d\Pi_f\,\mathcal{M}^*(p_1p_2\to f)\,\mathcal{M}(k_1k_2\to f)}$$

a b b a

SUMA PO SVIM MOGUĆIM FINALNIM STANJIMA

Ako je $|k_1k_2\rangle = |p_1p_2\rangle$ TJ. $a=b$ (FORWARD scattering)

$$2\,\mathrm{Im}\,\mathcal{M}(a\to a)=\sum_f\int d\Pi_f\,|\mathcal{M}(a\to f)|^2$$



$$\boxed{\mathrm{Im}\,\mathcal{M}(k_1k_2\to k_1k_2)=2E_{CM}\,p_{CM}\,\sigma_{TOT}(k_1k_2\to \text{anything})}$$

укупна енергија у CM

IMPULS ČESTICE U CM

IMAGINARNI DEO FORWARD SCATT. AMPLITUDE je proporcion SA PRESEKOM ZA PRODUKCIJU SVIH FINALNIH STANJA (OPT. TEOR)

Imaginarni deo amplitude M JE $\neq 0$ KADA SU VIRTUELNE čestice on-shell

$$\frac{1}{k^2-m^2+i\varepsilon}=P.\frac{1}{k^2-m^2}-\underbrace{i\pi\,\delta(k^2-m^2)}$$

$\mathcal{M}=\mathcal{M}(s)$; $s=E_{CM}^2=(p+p')^2=(k+k')^2$

$\mathcal{M}(s)$ je analitička fja kompleksne varijable s

$s_0\in\mathbb{R}$ → PRAG ZA PROIZVODNJU NAJLAKŠIH ČESTICA

ZA $\mathbb{R}\ni s<s_0 \underset{O.T}{\Rightarrow} \mathcal{M}(s)\in\mathbb{R}$ TJ.

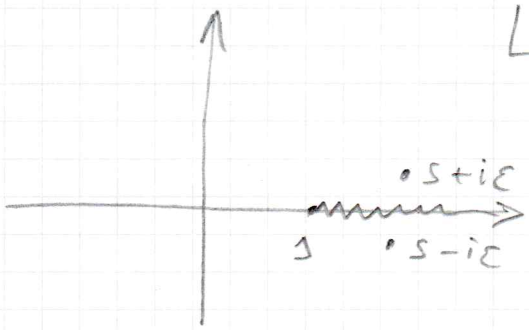
$$(\mathcal{M}(s^*))^*=\mathcal{M}(s)$$

ako je $\mathcal{M}(s)$ anal. fja $\Rightarrow$ $\mathcal{M}^*(s^*)$ je analit. fja (ZBOG Koši-Rimanovih uslova)

NAKON ANALITIČKOG PRODUŽENJA VAŽI:

$$\mathcal{M}(s) = \left(\mathcal{M}(s^*)\right)^*$$

[1]



$$\mathrm{Re}\,\mathcal{M}(s+i\varepsilon) + i\,\mathrm{Im}\,\mathcal{M}(s+i\varepsilon) = \mathrm{Re}\,\mathcal{M}(s-i\varepsilon) - i\,\mathrm{Im}\,\mathcal{M}(s-i\varepsilon)$$

$$\Downarrow$$

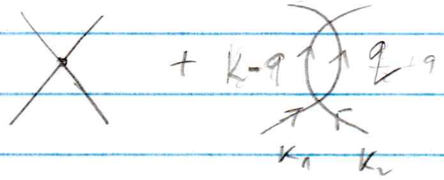
$$\boxed{\begin{aligned} \mathrm{Re}\,\mathcal{M}(s+i\varepsilon) &= \mathrm{Re}\,\mathcal{M}(s-i\varepsilon) \\ \mathrm{Im}\,\mathcal{M}(s+i\varepsilon) &= -\mathrm{Im}\,\mathcal{M}(s-i\varepsilon) \end{aligned}}$$

Diskontinuitet amplitude kroz zasek je

$$\begin{aligned} \mathrm{Disc}\,\mathcal{M}(s) &= \mathcal{M}(s+i\varepsilon) - \mathcal{M}(s-i\varepsilon) \\ &= 2i\,\mathrm{Im}\,\mathcal{M}(s+i\varepsilon) \end{aligned}$$

Preskripcija $+i\varepsilon$ u Feyn. dijagramima odgovara izračunavanju $\mathcal{M}$ iznad cut-a - tj u $1+i\varepsilon$

Izračunavanje imaginarnog dela amplitude u φ^4 teoriji u drugom redu po λ



$$iM = -i\lambda + i\delta M$$

$$k = k_1 + k_2 = (k^0, \vec{0}) \quad cm$$

$$i\delta M = \frac{\lambda^2}{2}\int \frac{d^4q}{(2\pi)^4}\,\frac{1}{q^2-m^2+i\varepsilon}\,\frac{1}{(k-q)^2-m^2+i\varepsilon}$$

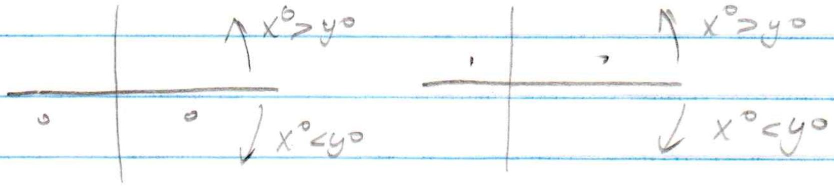
$$= \frac{\lambda^2}{2}\int \frac{d^4l_1}{(2\pi)^4}\frac{d^4l_2}{(2\pi)^4}(2\pi)^4\delta(l_1+l_2-k)\,\frac{1}{l_1^2-m^2+i\varepsilon}\,\frac{1}{l_2^2-m^2+i\varepsilon}$$

$$= \frac{\lambda^2}{2}\int \frac{d^4l_1}{(2\pi)^4}\frac{d^4l_2}{(2\pi)^4}(2\pi)^4\delta(l_1+l_2-k)\left[P\frac{1}{l_1^2-m^2} - i\pi\delta(l_1^2-m^2)\right]\left[P\frac{1}{l_2^2-m^2} - i\pi\delta(l_2^2-m^2)\right]$$

jer je $\boxed{\frac{1}{z\pm i\varepsilon} = P\left(\frac{1}{z}\right) \mp i\pi\delta(z)}$

$$\mathrm{Im}\, M = -\frac{\lambda^2}{2}\int \frac{d^4l_1}{(2\pi)^4}\frac{d^4l_2}{(2\pi)^4}(2\pi)^4\delta(l_1+l_2-k)\left[P\frac{1}{l_1^2-m^2}\,P\frac{1}{l_2^2-m^2} - \pi^2\delta(l_1^2-m^2)\,\delta(l_2^2-m^2)\right] \quad *$$

$$\int \frac{d^4l_1}{(2\pi)^4}\frac{d^4l_2}{(2\pi)^4}(2\pi)^4\delta(l_1+l_2-k)\,\underbrace{\frac{1}{l_1^2-m^2+i\varepsilon\,\mathrm{sgn}\,l_1^0}}_{\Delta_{ret}(l_1)}\,\underbrace{\frac{1}{l_2^2-m^2-i\varepsilon\,\mathrm{sgn}\,l_2^0}}_{\Delta_{adv}(l_2)} \equiv 0 \;!!$$



$$\int \frac{d^4 l_1}{(2\pi)^4} \frac{d^4 l_2}{(2\pi)^4} (2\pi)^4 \delta(l_1+l_2-k) \left[P\left(\frac{1}{l_1^2-m^2}\right) P\left(\frac{1}{l_2^2-m^2}\right) + \pi^2 \, \mathrm{sgn}\, l_{10} \, \mathrm{sgn}\, l_{20} \, \delta(l_1^2-m^2) \cdot \delta(l_2^2-m^2)\right] = 0 \quad **$$

KOMBINOVANJEM (*) i (**) $\Rightarrow$

$$\mathrm{Im}\, \mathcal{M} = \frac{\lambda^2}{2} \int \frac{d^4 l_1}{(2\pi)^4} \frac{d^4 l_2}{(2\pi)^4} (2\pi)^4 \delta(l_1+l_2-k) \, \pi^2 \delta(l_1^2-m^2) \, \delta(l_2^2-m^2) \underbrace{\left[1 + \mathrm{sgn}\, l_{10} \, \mathrm{sgn}\, l_{20}\right]}$$

$$= \begin{cases} 2, & l_{10}, l_{20} \text{ istog znaka} \\ 0, & \text{-''- suprotnog znaka} \end{cases}$$

SA DRUGE STRANE $l_1^0 + l_2^0 = k_0 > 0 \Rightarrow \underline{l_1^0 > 0 \wedge l_2^0 > 0}$

$$\mathrm{Im}\, \mathcal{M} = 2 \frac{\lambda^2}{2} \int \frac{d^4 l_1}{(2\pi)^4} \frac{d^4 l_2}{(2\pi)^4} (2\pi)^4 \delta(l_1+l_2-k) \, \pi^2 \delta(l_1^2-m^2) \, \theta(l_{10}) \, \delta(l_2^2-m^2) \, \theta(l_{20})$$

$$\Rightarrow \mathrm{Disc}\, \mathcal{M} = 2i \, \mathrm{Im}\, \mathcal{M}(s+i\varepsilon)$$

$$= 2i\lambda^2 \int \frac{d^4 q}{(2\pi)^4} \pi^2 \underbrace{\delta(q^2-m^2)\, \theta(q_0)}_{\frac{1}{2E_q}\delta(q^0-E_q)} \underbrace{\delta((k-q)^2-m^2)\, \theta(k^0-q^0)}_{\frac{1}{2E_q}\delta(k^0-q^0-E_q)}$$

$$= \frac{i\lambda^2}{32\pi^2} \int \frac{d^3 q}{E_q^2} \delta(k^0 - 2E_q)$$

$$= \frac{i\lambda^2}{32\pi^2} 4\pi \int_m^\infty \frac{q E_q \, dE_q}{E_q^2} \delta(2E_q - k^0)$$

$$\boxed{= \frac{i\lambda^2}{8\pi} \frac{1}{k^0} \sqrt{\left(\frac{k_0}{2}\right)^2 - m^2} \; \theta\left(\frac{k_0}{2} - m\right)}$$

Ovo je u skladu SA $\boxed{\mathrm{Im}\, \mathcal{M} = 2E_{cm} |\vec{p}| \sigma}$

$$\sigma = \frac{\lambda^2}{64\pi E_1^2} = \frac{\lambda^2 \cdot 4}{64\pi \, k_0^2}$$

$$2E_{cm} |\vec{k}| \sigma = 2 \cancel{k_0} \sqrt{\left(\frac{k_0}{2}\right)^2 - m^2} \cdot \frac{\lambda^2}{16\pi k_0^{\cancel{2}}}$$

$$\equiv \mathrm{Im}\, \mathcal{M}$$

Cutkosky - rule (cutting rule)

ZA NALAZENJE DISKONTINUITETA Feynmanovog dijagrama

1) Cut the diagram in all possible ways such that the cut propagators are on shell without violating conservation of momenta

2) For each cut. $\dfrac{1}{p^2-m^2+i\varepsilon} \longrightarrow -2\pi i\,\delta^{(4)}(p^2-m^2)\,\theta(p^0)$

3) sum the contributions of all possible cuts

$$\mathrm{Disc}\,\mathcal{M} = -\frac{i\lambda^2}{2}\int \frac{d^4q}{(2\pi)^4}\,(-2\pi i)^2\,\delta(q^2-m^2)\,\theta(q^0)\,\delta((k-q)^2-m^2)\,\theta(k^0-p^0)$$

$$= -\frac{i\lambda^2}{2}\int \frac{d^4p_1}{(2\pi)^4}\int \frac{d^4p_2}{(2\pi)^4}\,\delta(p_1+p_2-k)\,(2\pi)^4\,(-2\pi i)^2\,\delta(p_1^2-m^2)$$

$$\cdot\,\delta(p_2^2-m^2)\,\theta(p_{10})\,\theta(p_{20})$$

$$= \frac{i}{2}\int \frac{d^3p_1}{(2\pi)^3}\int \frac{d^3p_2}{(2\pi)^3}\,\frac{1}{2E_1}\,\frac{1}{2E_2}\,(2\pi)^4\,\delta(p_1+p_2-k)\,|i\lambda|^2$$

$$= 2i\,2E\,|\vec{k}|\cdot\sigma$$

NESTABILNE ČESTICE

 $= i\Delta(p^2) = \dfrac{i}{p^2 - m_0^2 - \Pi(p^2) + i\varepsilon}$

 $= -i\Pi(p^2)$

Ako $\Pi(p^2)$ ima imaginarni deo onda moramo pažljivo definisati $\underline{m}$ i $\underline{Z}$. Ako je $\mathrm{Im}\,\Pi(p^2) \neq 0 \Rightarrow$ da je čestica nestabilna!

$$p^2 - m_0^2 - \mathrm{Re}\,\Pi(p^2) = p^2 - m_0^2 - \mathrm{Re}\,\Pi(m^2) - \mathrm{Re}\,\Pi'(m^2)(p^2 - m^2) - \sum_{i\geq 2} c_i (p^2 - m^2)^i - i\,\mathrm{Im}\,\Pi(p^2)$$

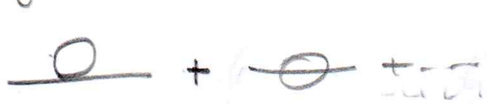
$$\boxed{m_0^2 - \mathrm{Re}\,\Pi(p^2) = m^2}$$ → FIZIČKA MASA

$$i\Delta(p^2) = \frac{iZ}{(p^2 - m^2)\left(1 - \dfrac{\sum_{i\geq 1} c_i (p^2 - m^2)^i}{1 - \mathrm{Re}\,\Pi'(m^2)} - \dfrac{i\,\mathrm{Im}\,\Pi(p^2)}{(p^2 - m^2)(1 - \mathrm{Re}\,\Pi'(m^2))}\right)}$$

$$\boxed{Z = \frac{1}{1 - \mathrm{Re}\,\Pi'(m^2)}}$$

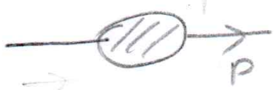
$$i\Delta(p^2) \sim \frac{iZ}{p^2 - m^2 - iZ\,\mathrm{Im}\,\Pi(p^2) + i\varepsilon} \qquad (p^2 \approx m^2) \text{ u blizini pola}$$

- AMPLITUDA ZA "RASEJANJE" $1 \to 1$ tj. čestice $p \to p$ je

$$i\mathcal{M}(p \to p) = Z\,\big($$  $$\big)$$
$$= -iZ\,\Pi(p^2)\big|_{p^2 = m^2}$$

- OPTIČKI TEOREM: $\mathrm{Im}\,\mathcal{M}(p \to p) = m\,\Gamma(p \to \text{bilo šta})$

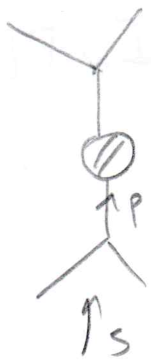
$$\Rightarrow \boxed{-Z\,\mathrm{Im}\,\Pi(p^2)\big|_{p^2 \approx m^2} = m\Gamma}$$



$$= i\Delta(p^2) = \frac{iZ}{p^2 - m^2 - iZ\,\mathrm{Im}\,\Pi(p^2) + i\varepsilon} \qquad (p^2 \approx m^2)$$

$$= \frac{iZ}{p^2 - m^2 + im\Gamma}$$

ako je ovaj propagator u S-kanalu F-dij $\Rightarrow$



$$\sigma \sim \left| \frac{1}{s - m^2 + im\Gamma} \right|^2$$

Breit-Wignerova formula

$$\tau \sim \frac{1}{\Gamma}$$