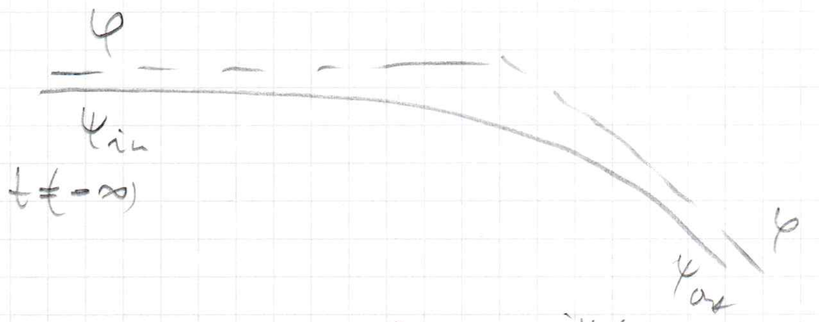
S MATRICA (VIDI KTP1)

<u>in</u> i <u>out</u> STANJA

$$|\Psi_\alpha, in\rangle \equiv |\Psi_\alpha(0)\rangle = \lim_{t\to-\infty} e^{iHt} |\Psi_\alpha(t)\rangle \rightarrow \text{evoluirajući po } H$$

$$H = H_0 + \underbrace{H_{int}\, e^{-\alpha|t|}}_{u\ t\to\pm\infty}$$

iskljucujemo interakciju



$$|\varphi_\alpha\rangle = \lim_{t\to-\infty} e^{iH_0 t} |\varphi_\alpha(t)\rangle$$

$$\Rightarrow \lim_{t\to-\infty} e^{-iHt}\Psi_{in}(0) = \lim_{t\to-\infty} e^{-iH_0 t}\varphi(0)$$

$$\Rightarrow |\Psi_{in}\rangle = \lim_{t\to-\infty} e^{iHt} e^{-iH_0 t} |\varphi\rangle \equiv \Omega_+ |\varphi\rangle$$

$$= U(0,-\infty)|\varphi\rangle$$

<u>slično</u>: $$\lim_{t\to\infty} e^{-iHt}\Psi_{out}(0) = \lim_{t\to\infty} e^{-iH_0 t}\varphi(0)$$

$$|\Psi_{out}\rangle = \lim_{t\to\infty} e^{iHt} e^{-iH_0 t}|\varphi\rangle$$

$$= U_I(0,\infty)|\varphi\rangle$$

$$\equiv \Omega_- |\varphi\rangle$$

$$S_{\beta\alpha} = \langle \Psi_\beta\, out | \Psi_\alpha\, in\rangle = \langle \varphi_\beta | \Omega_-^{-1} \Omega_+ | \varphi_\alpha\rangle =$$

$$= \langle \varphi_\beta | U_I(\infty,-\infty) | \varphi_\alpha\rangle$$

$\langle \Psi_I(\infty)|$ evol. oper u int. slici $|\Psi_I(-\infty)\rangle$

S MATRICA U INTERAKCIONOJ SLICI

$$S = \Omega_-^{-1}\Omega_+ = U_I(\infty, -\infty)$$

In i Out stanja su Hajzenbergova stanja

$\simeq$ IZOLOVANI TALASNI PAKETI SKORO slobodnih čestica u $\pm\infty$

$\phi_{in}(x)$, $\phi_{out}(x) \rightarrow$ HAJZENBERGOVA POLJA KOJA KREIRAJU jednočestična stanja delovanjem na $|\Omega\rangle$

$$(\Box + m^2)\phi_{in/out} = 0$$

$$\phi(x) \xrightarrow[t\to\pm\infty]{} \sqrt{Z}\,\phi_{in/out}$$ VAŽI NA STANJIMA

$\phi(x)$ KADA deluje na vakuum kreira i višečestična stanja, $\phi_{in/out}$ kreiraju samo jednočestična stanja

$$\phi_{in/out}(x) = \int \frac{d^3k}{(2\pi)^{3/2}\sqrt{2E_k}}\left(a_{k\,in/out}\,e^{-ik\cdot x} + a^+_{k\,in/out}\,e^{ikx}\right)$$

$$|\vec{k}_1, \dots \vec{k}_n, in\rangle = a^+_{k_1 in} \dots a^+_{k_n in}|\Omega\rangle$$
$$|\vec{k}_1, \dots \vec{k}_m, out\rangle = a^+_{k_1\,out} \dots a^+_{k_m out}|\Omega\rangle$$

$$S_{\beta\alpha} = \langle \Psi_{\beta\,out}|\Psi_{\alpha\,in}\rangle = \langle \Psi_{\beta\,out}|S|\Psi_{\alpha\,out}\rangle$$
$$= \langle \Psi_{\beta\,in}|S|\Psi_{\alpha\,in}\rangle$$

$$\Rightarrow |\Psi_{\alpha}, in\rangle = S\,|\Psi_{\alpha\,out}\rangle$$

S-MATRICA u Hajz. slici!

$$a^+_{in}(\vec{k})\,|\alpha, in\rangle = a^+_{in}(\vec{k})\, S\, |\alpha, out\rangle$$
$$= S\, a^+_{out}(\vec{k})\, |\alpha, out\rangle$$

$$\Rightarrow \quad \boxed{S^{-1} a^+_{in}(\vec{k})\, S = a^+_{out}(\vec{k})}$$

ODAVDE je:

$$\boxed{\phi_{out}(x) = S^{-1}\, \phi_{in}(x)\, S}$$

in i out polja su unitarno ekvivalentni.

$$S = \sum_\lambda |\lambda\ in\rangle\langle\lambda\ out| = \sum_\lambda \Omega_+ |\varphi_\lambda\rangle\langle\varphi_\lambda| \Omega_-^{-1}$$

$$\Downarrow$$

$$\boxed{S = \Omega_+ \Omega_-^{-1}}$$

$$\phi_{in}(x) = U(t, -\infty)\, \phi(x)\, U^{-1}(t, -\infty)$$
$$\phi_{out}(x) = U(t, +\infty)\, \phi(x)\, U^{-1}(t, +\infty)$$

$$\boxed{\phi(x) \xrightarrow[t\to\pm\infty]{} \sqrt{Z}\, \phi_{out,in}(x)}$$

Vazi na matrični element (weak converge)

$$\langle a|\, \phi \,|b\rangle \to \sqrt{Z} \langle a|\, \phi_{in,out} \,|b\rangle$$

$$\phi_{in,out} = \int \frac{d^3\bar{p}}{(2\pi)^{3/2}} \frac{1}{\sqrt{2E_p}} \left(a_{p\,in,out}\, e^{-ipx} + a^{+}_{p\,in,out}\, e^{ipx} \right)$$

$$S_{fi} = \langle \bar{q}_1 \ldots \bar{q}_m, out \,|\, \bar{p}_1 \ldots \bar{p}_n, in \rangle$$

$$= \langle \bar{q}_1, \ldots \bar{q}_m, out |\, a^{+}_{p_1 in} \,| \bar{p}_2, \ldots \bar{p}_n, in\rangle$$

$$= \langle \bar{q}_1 \ldots \bar{q}_m, out |\, a^{+}_{p_1 out} \,| \bar{p}_2, \ldots \bar{p}_n, in\rangle \sim \delta(\vec{p}_1 - \bar{q}_i)$$

$$+ \langle \bar{q}_1, \ldots \bar{q}_m, out |\, a^{+}_{p_1 in} - a^{+}_{p_1 out} \,| \bar{p}_2, \ldots \bar{p}_n, in\rangle$$

PROBLEM 7.1

$$a^{+}_{p\,in} = \frac{i}{(2\pi)^{3/2}\sqrt{2E_p}} \int d\bar{x}\; \phi_{in} \overleftrightarrow{\partial_0} e^{-ipx}$$

$$a \overleftrightarrow{\partial_0} b \stackrel{def}{=} a\partial_0 b - \partial_0 a \cdot b$$

$$a^{+}_{p\,in} - a^{+}_{p,out} = \frac{i}{(2\pi)^{3/2}\sqrt{2E_p}} \int d\vec{x} \left(\phi_{in} - \phi_{out}\right) \overleftrightarrow{\partial_0} e^{-ipx}$$

$$= \frac{i}{(2\pi)^{3/2}\sqrt{2E_p}} \int d^3\vec{x} \left[\lim_{x^0\to-\infty} \phi_{in} \overleftrightarrow{\partial_0} e^{-ipx} - \lim_{x^0\to\infty} \phi_{out} \overleftrightarrow{\partial_0} e^{-ipx} \right]$$

$$= \frac{-i}{(2\pi)^{3/2}\sqrt{2E_p}} \frac{1}{\sqrt{Z}} \int d^3x \left(\lim_{x_0\to\infty} - \lim_{x_0\to-\infty} \right) \phi(x) \overleftrightarrow{\partial_0} e^{-ipx}$$

$$= -\frac{i}{(2\pi)^{3/2}\sqrt{2E_p}} \frac{1}{\sqrt{Z}} \int d^4x\; \partial_0 \Big[\underbrace{\phi(x) \overleftrightarrow{\partial_0} e^{-ipx}}_{\phi \partial_0 e^{-ipx} - e^{-ipx} \partial_0 \phi} \Big]$$

$$= -\frac{i}{(2\pi)^{3/2}\sqrt{2E_p}} \frac{1}{\sqrt{Z}} \int d^4x \left(\phi\, \partial_0^2 e^{-ipx} - e^{-ipx} \partial_0^2 \phi \right)$$

$$= \frac{i}{(2\pi)^{3/2}\sqrt{2E_p}} \frac{1}{\sqrt{Z}} \int d^4x \, e^{-ipx} (\Box_x + m^2)\, \phi(x) \quad \checkmark$$

$$S_{fi} = \frac{i}{(2\pi)^{3/2}\sqrt{2E_p}} \frac{1}{\sqrt{Z}} \int d^4x \, \langle \vec{q}_1, \ldots \vec{q}_m, out | \hat{\phi}(x) | \vec{p}_2 \ldots \vec{p}_n, in \rangle (\overleftarrow{\Box_x} + m^2)\, e^{-ip_1 x}$$

$$\phi(x)\, a^+_{p_2, in} = \frac{i}{(2\pi)^{3/2}\sqrt{2E_{p_2}}} \int d\vec{y} \; \phi(x)\, \phi_{in}(y) \overleftrightarrow{\partial}_{y_0} e^{-ip_2 y} \Big|_{y_0 = -\infty}$$

$$= \frac{i}{(2\pi)^{3/2}\sqrt{2E_{p_2}}} \frac{1}{\sqrt{Z}} \lim_{y_0 \to -\infty} \int d\vec{y} \; \phi(x)\, \phi(y) \overleftrightarrow{\partial}_{y_0} e^{-ip_2 y}$$

$$= \frac{i}{(2\pi)^{3/2}\sqrt{2E_{p_2}}} \frac{1}{\sqrt{Z}} \lim_{y_0 \to -\infty} \int d\vec{y} \; T(\phi(x)\, \phi(y)) \overleftrightarrow{\partial}_{y_0} e^{-ip_2 y}$$

$$= \frac{i}{(2\pi)^{3/2}\sqrt{2E_{p_2}}} \frac{1}{\sqrt{Z}} \Big[\lim_{y_0 \to \infty} \int d\vec{y} \; T(\phi(x)\, \phi(y)) \overleftrightarrow{\partial}_{y_0} e^{-ip_2 y}$$

$$- \int d^4y \; \partial_{y_0} \left(T(\phi(x)\, \phi(y)) \overleftrightarrow{\partial}_{y_0} e^{-ip_2 y} \right)$$

I član $= a^+_{p_2, out}\, \phi(x)$ ali $\vec{p}_2 \neq \vec{q}_i$ pa nema doprinosa (?)

$$\Rightarrow \phi(x)\, a^+_{p_2, in} = \frac{i}{(2\pi)^{3/2}\sqrt{2E_{p_2}}} \frac{1}{\sqrt{Z}} \int d^4y \; e^{-ip_2 y} (\Box_y + m^2)\, T(\phi(x)\, \phi(y))$$

<u>Дакле</u>:

$$S_{fi} = \left(\frac{i}{\sqrt{Z}}\right)^{n+m} \int d^4y_1 \ldots d^4y_m \, d^4x_1 \ldots d^4x_n \; u^*_{q_1}(y_1) \ldots u^*_{q_m}(y_m) \cdot u_{p_1}(x_1) \ldots u_{p_n}(x_n) \cdot$$

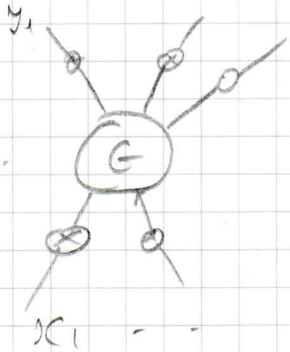
$$(\Box_{x_1} + m^2) \ldots \ldots (\Box_{y_m} + m^2) \langle \Omega | T(\phi(y_1) \ldots \phi(x_n)) | \Omega \rangle$$

gde je $u_p(x) = \dfrac{e^{-ipx}}{(2\pi)^{3/2}\sqrt{2E_p}}$

LSZ

S-matrični element za rasejanje n-bozona u m bozona dobija se iz odgovarajuće $n+m$-Grinove f-je zamenom eksternih propagatora sa $\frac{+i}{\sqrt{Z}} u_p(x)$ za ulazne odnosno $\frac{+i}{\sqrt{Z}} u_p^*(x)$ za izlazne

$$S_{fi} = \left(\frac{1}{\sqrt{Z}}\right)^{n+m} \cdots$$



(ампутирамо спољне пропагаторе)

Прелазак у impulsni prostor

$$S_{fi} = \left(\frac{i}{\sqrt{Z}}\right)^{n+m} \int dy_1 \dots dy_m \, dx_1 \dots dx_n \; N_{q_1} \dots \; N_{p_n}$$

$$\times \prod e^{i q_i y_i} \prod e^{-i p_j x_j} \left(\Box_{y_1} + m^2\right) \dots \left(\Box_{x_n} + m^2\right) \times$$

$$\langle \Omega | T (\phi(y_1) \dots \phi(x_n) | \Omega \rangle$$

$$= \left(\frac{-i}{\sqrt{Z}}\right)^{n+m} \int dy_1 \dots dx_1 \dots dx_n \; N_{q_1} \dots N_{p_n} \, e^{i q_1 y_1} \dots e^{i q_m y_m} e^{-i p_1 x_1} \dots$$

$$(q_1^2 - m^2) \dots (q_m^2 - m^2)\,(p_1^2 - m^2) \dots (p_n^2 - m^2)$$

$$\circ \langle \Omega | T\phi(y_1) \dots \phi(x_n) | \Omega \rangle$$

PRIMEDBA: S-funkcional je

$$S = :e^{\int dy \left[\phi_{in}(y) \left((\Box_y + m^2) \frac{1}{\sqrt{Z}} \frac{\delta}{\delta J(y)} \right) \right]}: Z(J) \Big|_{J=0}$$

$$G^{(2)}(x,y) = \langle \Omega | T(\phi(x)\,\phi(y)) | \Omega \rangle$$

PRELAZAK U IMPULSNI PROSTOR

$$\int dx\,dy\; e^{ipx + iqy} \langle \Omega | T(\phi(x)\,\phi(y)) | \Omega \rangle = (2\pi)^4\, \delta(p+q)\; G(p,-p)$$
$$= (2\pi)^4\, \delta(p+q)\; i\Delta(p)$$

$i\Delta(p) =$  $=$ Full propagator

1PI — JEDNOČESTIČNI IRED. DIO

$= -i\Pi(p^2)$


$$= \frac{i}{p^2 - m_0^2 + i\varepsilon} + \frac{i}{p^2 - m_0^2 + i\varepsilon}(-i\Pi)\frac{i}{p^2 - m_0^2 + i\varepsilon} + \dots$$

$$= \frac{i}{p^2 - m_0^2 + i\varepsilon}\left(1 + \Pi(p^2)\frac{1}{p^2 - m_0^2 + i\varepsilon} + \Pi(p^2)\frac{1}{p^2 - m_0^2 + i\varepsilon}\,\Pi(p^2)\frac{1}{p^2 - m_0^2 + i\varepsilon} + \dots\right)$$

$$= \frac{i}{p^2 - m_0^2 + i\varepsilon}\;\frac{1}{1 - \dfrac{\Pi(p^2)}{p^2 - m_0^2 + i\varepsilon}} = \frac{i}{p^2 - m_0^2 - \Pi(p^2) + i\varepsilon}$$

$m \rightarrow$ FIZIČKA MASA

$m_0 \rightarrow$ GOLA MASA

$$\Pi(p^2) = \underbrace{\Pi(m^2)}_{\text{kvad. div}} + \underbrace{\frac{d\Pi}{dp^2}\Big|_{m^2}}_{\text{log div}}(p^2 - m^2) + \underbrace{\tilde{\Pi}(p^2)}_{\text{konačan}} \quad \sim (p^2 - m^2)^2(\dots)$$

$$i\Delta(p) = \frac{i}{p^2 - m_0^2 - \Pi(m^2) - (p^2 - m^2)\,\Pi'(m^2) - \tilde{\Pi}(p^2)} = \frac{i}{(p^2 - m^2)(1 - \Pi'(m^2)) - \tilde{\Pi}(p^2)}$$

$$\breve{\Pi}(p^2) = (1 + \Pi'(m^2))\,\tilde{\Pi}(p^2) + \mathcal{O}(\lambda_0^2)$$

$$\Rightarrow \boxed{i\Delta(p) = \frac{iZ}{p^2 - m^2 - \frac{\tilde{\Pi}(p^2)}{1-\Pi'(m^2)}}}$$

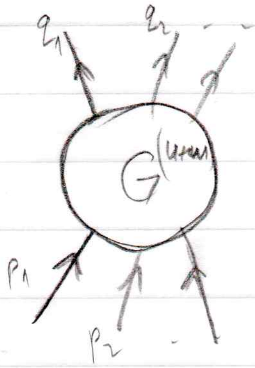
gdje je $Z = \dfrac{1}{1-\Pi'(m^2)} \approx \underline{\underline{1 + \Pi'(m^2)}}$

Propagator ima pol u $p^2 = m^2$; REZIDUUM JE $\underline{\underline{Z}}$

$$\boxed{i\Delta(p) \underset{p^2=m^2}{\sim} \frac{iZ}{p^2-m^2} + \text{reg. dio}}$$

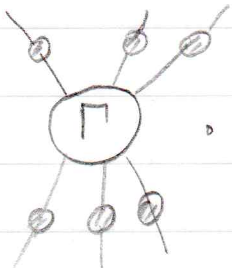
$$\boxed{\begin{aligned} S_{fi} = \left(\frac{+i}{\sqrt{Z}}\right)^{n+m} N_{q_1} \ldots N_{p_1} (q_1^2 - m^2) \ldots (p_n^2 - m^2) (2\pi)^4 \delta^{(4)}(p_1 + \ldots p_n - q_1 \ldots - q_m) \\ \times\, G^{(n+m)}(q_1, \ldots q_m, -p_1, \ldots -p_n) \end{aligned}}$$

LSZ



$$= \prod_{i=1}^{n} \frac{iZ}{p_i^2 - m^2 - \tilde{\Pi}(p^2) + i\varepsilon} \prod_{j=1}^{m} \frac{iZ}{q_j^2 - m^2 - \tilde{\Pi}(q^2) + i\varepsilon} \underbrace{\Gamma^{(n+m)}(p_1, \ldots)}_{\text{amputirana G. f.}}$$

(ovo je definicija)

$=$ 

$$\cdot \prod \frac{iZ}{p_i^2 - m^2 + \tilde{\Pi}(p^2)} \prod_j \frac{iZ}{q_j^2 - m^2 - \tilde{\Pi}(q_j^2)}$$

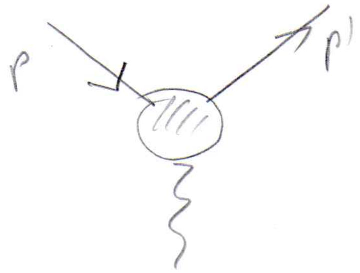
$$S_{fi} = \left(\frac{+i}{\sqrt{Z}}\right)^{n+m} N_{p_1} \ldots N_{q_m} (p_1^2 - m^2) \ldots (q_m^2 - m^2) \frac{(iZ)^{n+m}}{(p_1^2 - m^2 - \Pi) \ldots (q_m^2 - m^2 + \Pi)}$$

$$\times\, \Gamma^{(n+m)}(p_1, \ldots) \cdot (2\pi)^4 \delta(p_1 + \ldots p_n - q_1 - \ldots q_m)$$

$$\xrightarrow[\substack{p_i^2 = m^2 \\ q_j^2 = m^2}]{} \boxed{(\sqrt{Z})^{n+m} N_{q_1} \ldots N_{p_m} \Gamma^{(n+m)}(p_1, \ldots) \cdot (2\pi)^4 \delta(p_1 + p_2 - q_1 - q_2)}$$

(on shell)

VRAĆAMO SE NA VERTEKSNU dij. u QED



$$\Gamma^\mu(p',p) \longrightarrow \overbrace{\sqrt{Z_2}\,\sqrt{Z_2}}^{\text{renormalizacijski faktori (potiču od self energ.)}}\,\Gamma^\mu(p',p)\,\underbrace{(\sqrt{Z_3})}_{\text{nebitno sada jer foton nije spolj. linija}}$$

$$\underbrace{Z_2\,\Gamma^\mu}_{\text{VERT. dij.}} = \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)$$

$$Z_2\,\Gamma^\mu = (1+\delta Z_2)(\gamma^\mu + \delta\Gamma^\mu) = \gamma^\mu + \underbrace{\delta\Gamma^\mu + \gamma^\mu\delta Z_2}_{\text{KOREKCIJE}}$$

$$= \gamma^\mu + \gamma^\mu \overset{\text{STARO}}{F_1}(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}\overset{\text{STARO}}{F_2}(q^2) + \gamma^\mu\delta Z_2$$

vidimo da $F_1(q^2) \to F_1(q^2) + \delta Z_2$

$$= F_1(q^2) - \delta F_1(0)$$