ZAKOČNO ZRAČENJE (BREMSSTRAHLUNG)

Klasično:

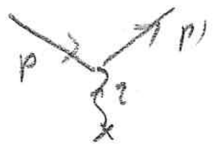
NAELEKTRISANE čestice gube energiju pri ubrzanom kretanju

$\Leftrightarrow$ kočenje zračenje

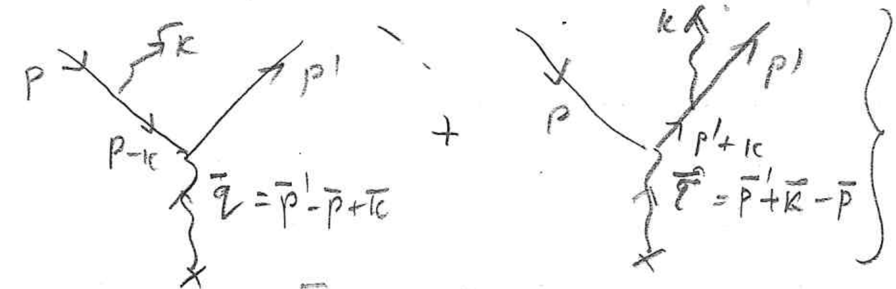
Kvantno: dolazi do emisije fotona.

Rasejanje e^- na Kulonovom potencijalu

U realnom eksperimentu detektor ima meku rezoluciju energije ΔE, i ne možemo reći da li je pored rasejanja elektrona imamo i emitovani realan i mek foton



$$\left.\begin{aligned} S_{(0)} &= \frac{m}{V\sqrt{E_i E_f}}\, 2\pi\, \delta(E_f - E_i)\cdot i\mathcal{M}_0 \\ i\mathcal{M}_0 &= -ie\, \bar{u}(p')\gamma^\mu u(p)\, \tilde{A}_\mu(\vec{q}) \end{aligned}\right\} \text{elastično rasejanje}$$



emituje se meki foton impulsa k

$$i\mathcal{M} = -ie^2\, \bar{u}(p')\left[\tilde{A}(\vec{q})\, \frac{\not{p}-\not{k}+m}{(p-k)^2-m^2}\, \not{\varepsilon}^*(k) + \not{\varepsilon}^*(k)\, \frac{\not{p}'+\not{k}+m}{(p'+k)^2-m^2}\, \tilde{A}(\vec{q})\right] u(p)$$

$$S_{(1)} = \frac{m}{V\sqrt{E_i E_f}}\, \frac{1}{\sqrt{2V\omega_k}}\, 2\pi\, \delta(E_f + \omega_k - E_i)\cdot i\mathcal{M}$$

Zanima nas emisija mekog fotona $\omega_k \approx 0$, tj $|\vec{p}| \approx |\vec{p}'|$ odnosno $|\vec{p}' - \vec{p}| \gg |\vec{k}|$

U brojiocu ćemo zanemariti impuls fotona, a u imeniocu uvodimo fiktivnu masu fotona μ, tj

$$k^2 = \mu^2 \ll m^2 \ll |\vec{q}^2|$$

Primenom Dirakove jednačine dobijam

$$i\mathcal{M} = -ie^2\, \bar{u}(p') \left[\not{A}(q) \frac{2p\cdot\varepsilon^*(k)}{-2p\cdot k+\mu^2} + \frac{2p'\cdot\varepsilon^*(k)}{2p'k+\mu^2} \not{A}(q) \right] u(p)$$

$$= \underbrace{i\mathcal{M}_0}_{\text{amplituda za elastično rasejanje}} \cdot \underbrace{e\left[\frac{2p\cdot\varepsilon^*(k)}{-2p\cdot k+\mu^2} + \frac{2p'\varepsilon^*(k)}{2p'\cdot k+\mu^2} \right]}_{\text{faktor za emisija fotona}}$$

$$S_{fi} = 2\pi\,\delta(E_f+\omega_k-E_i) \sqrt{\frac{m}{VE_i}} \sqrt{\frac{m}{VE_f}} \frac{1}{\sqrt{2V\omega_k}}\, i\mathcal{M}_0\, e \left[\frac{2p\cdot\varepsilon^*}{-2p\cdot k+\mu^2} + \frac{2p'\varepsilon^*}{2p'k+\mu^2} \right]$$

Presek:

$$d\sigma_B = \frac{\langle |S_{fi}|^2 \rangle}{T} \frac{VE_i}{|\vec{p}|} \frac{Vd^3p_f}{(2\pi)^3} \frac{Vd^3k}{(2\pi)^3}$$

$$= 2\pi\,\underbrace{\delta(E_f+\omega_k-E_i)}_{\text{zanem.}} \frac{m^2 \cancel{V}E_i}{\cancel{V^2}E_iE_f|\vec{p}|} |i\mathcal{M}_0|^2 \frac{\cancel{V}d^3p_f}{(2\pi)^3} \frac{e^2}{2\cancel{V}\omega_k} \cdot \sum_r \left| \frac{2p\cdot\varepsilon}{-2p\cdot k+\mu^2} + \frac{2p'\varepsilon}{2p'k+\mu^2} \right|^2 \cdot \frac{\cancel{V}d^3k}{(2\pi)^3}$$

$$= \underbrace{(d\sigma)_0}_{\text{presek za elastično rasejanje}} \underbrace{e^2 \sum_r \left| \frac{2p\cdot\varepsilon(k)}{-2p\cdot k+\mu^2} + \frac{2p'\cdot\varepsilon(k)}{2p'k+\mu^2} \right|^2 \cdot \frac{d^3k}{(2\pi)^3\, 2\omega_k}}_{\text{za } \mu=0 \text{ i } \omega_k\approx 0 \text{ ovo je divergentno}}$$

Ova divergencija se javlja jer za $k\approx 0 \Rightarrow p^2 = p'^2 = m^2$ i tu su Dirakovi propagatori singularni

za $\mu = 0$ integral se ponaša kao $\int \frac{d\omega_k}{\omega_k} \Rightarrow$

IC katastrofa

SUMIRANJE PO STANJIMA POLARIZACIJE (MASENA TEM)

$$\sum_{r=1}^{3} \varepsilon_r^{\mu}(\vec{k})\, \varepsilon_r^{\nu}(\vec{k}) = -g^{\mu\nu} + \frac{k^{\mu}k^{\nu}}{\mu^2} \longrightarrow \text{ne daje doprinos}$$

$$\sum_r |\cdots|^2 = \frac{-4p^2 \simeq m^2}{(\mu^2 - 2p\cdot k)^2} + \frac{-4p'^2 \simeq m^2}{(\mu^2 + 2p'k)^2} - \frac{8\, p\cdot p'}{(\mu^2+2p'k)(\mu^2 - 2p\cdot k)}$$

① ② ③

$$\left.\begin{array}{l} q^2 = 2m^2 - 2p\cdot p' \\ |q^2| \gg m^2 \end{array}\right\} \Rightarrow |q^2| \approx |-2p\cdot p'| \gg m^2$$

i zato zanemarujem ① i ② u odnosu na ③

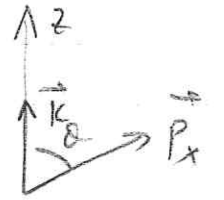
$$\sum_r |\cdots|^2 \approx \frac{2p\cdot p'}{(p\cdot k)(p'k)}$$

$$d\sigma_B = (d\sigma)_0 \,\frac{e^2\, 2p\cdot p'}{(k\cdot p)(kp')}\, \frac{d^3k}{(2\pi)^3\, 2\omega_k}$$

$$\Rightarrow d\sigma_0\, \frac{e^2 p\cdot p'}{(2\pi)^3} \int \frac{d|\vec{k}|\, \vec{k}^2}{\omega_k} \int d\Omega \,\frac{1}{(kp)(kp')}$$

$$\int d\Omega \,\frac{1}{(kp)(kp')} = \int_0^1 dx \int d\Omega\, \frac{1}{(k\cdot p_x)^2} \quad \text{gdje je}$$

$$p_x^{\mu} = x\,p^{\mu} + (1-x)\,p'^{\mu}$$



$$\longrightarrow \int_0^1 dx\; 2\pi \int_{-1}^{1} d(\cos\theta)\, \frac{1}{k^2\,(E_x - |\vec{p}_x|\cos\theta)^2}$$

$$k^{\mu} \approx (k, 0, 0, k)$$

$$p_x^{\mu} = (E_x, \vec{p}_x)$$

$$= \frac{2\pi}{k^2}\int_0^1 dx\, \frac{1}{|\vec{p}_x|}\left[\frac{1}{E_x - |\vec{p}_x|} - \frac{1}{E_x + |\vec{p}_x|}\right]$$

$$= \frac{4\pi}{k^2}\int_0^1 dx\, \frac{1}{p_x^2} = \frac{4\pi}{k^2}\int_0^1 \frac{dx}{m^2 - x(1-x)q^2}$$

F

$$\int_0^1 \frac{dx}{q^2\left(x^2 - x + \frac{m^2}{q^2}\right)} = \frac{1}{q^2}\int_0^1 dx \left[\frac{1}{x - 1 + \frac{m^2}{q^2}} - \frac{1}{x - \frac{m^2}{q^2}}\right]\frac{1}{1 - \frac{2m^2}{q^2}}$$

$$= \frac{1}{\cancel{q^2 - 2m^2}}\left(\ln\left|\frac{\frac{m^2}{q^2}}{\frac{m^2}{q^2} - 1}\right| - \ln\left|\frac{1 - \frac{m^2}{q^2}}{-\frac{m^2}{q^2}}\right|\right)$$

$$\approx \frac{2}{q^2}\ln\left|\frac{m^2}{m^2 - q^2}\right| \approx -\frac{2}{q^2}\ln\left|\frac{-q^2}{m^2}\right|$$

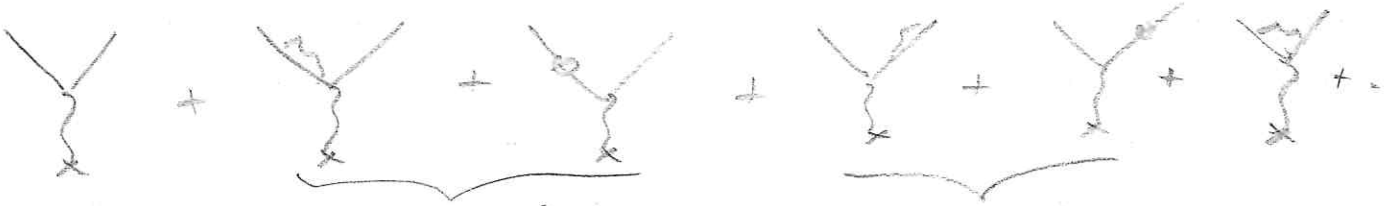
Dakle $d\sigma_B = d\sigma_0 \dfrac{e^2}{(2\pi)^3}\,\dfrac{-q^2}{2}\displaystyle\int \frac{4\pi}{k^2}\left(-\frac{2}{q^2}\right)\ln\left(-\frac{q^2}{m^2}\right)\frac{\vec{k}^2\, d|\vec{k}|}{\omega_k}$

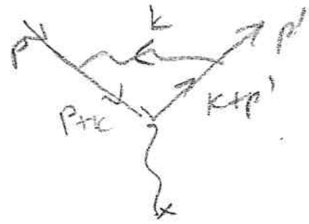
$$= d\sigma_0 \frac{e^2}{2\pi^2}\ln\left(-\frac{q^2}{m^2}\right)\int_\mu^{k_{max} = \sqrt{(\Delta E)^2 - \mu^2} \approx \Delta E} \frac{dk}{k}$$

$$\boxed{d\sigma_B = d\sigma_0 \frac{e^2}{4\pi^2}\ln\left(-\frac{q^2}{m^2}\right)\ln\left(\frac{k_{max}^2}{\mu^2}\right)}$$

divergira za $\mu \to 0$

IC divergencije u QED





$$= (-ie)^3 (-i^3) \int \frac{d^4k}{(2\pi)^4}\, \bar{u}(p')\, \gamma_\nu \frac{\not{k} + \not{p}' + m}{(k+p')^2 - m^2}\, \gamma^\mu \frac{\not{k} + \not{p} + m}{(k+p)^2 - m^2}\, \gamma^\nu \cdot \frac{1}{k^2 - \mu^2}$$

$\mu \rightarrow$ IC parametar

Zanemarujem lin. članove po k u brojioku. Zanima nas IC div. integrala $(k \to 0)$

$$\simeq -e^3 \int \frac{d^4k}{(2\pi)^4} \frac{4(p \cdot p')\, \bar{u}(p')\, \gamma^\mu u(p)}{(k^2 + 2kp')(k^2 + 2kp)(k^2 - \mu^2)} + \text{IC kon. deo}$$

$$|q^2| = |(p'-p)^2| \gg m^2 \gg \mu^2$$

Feynmanova parametrizacija

$$\frac{1}{(k^2 + 2kp)(k^2 + 2kp')} = \int_0^1 dx \frac{1}{(k^2 + 2P_x k)^2}$$

$$P_x = xp + (1-x)p'$$

$$\frac{1}{(k^2 - \mu^2)(k^2 + 2kp)(k^2 + 2kp')} = \int_0^1 dz\, 2z \int_0^1 dx \frac{1}{(k^2 - \mu^2 + (P_x k + \mu^2) z)^3}$$

$$= \int_0^1 dz\, 2z \int_0^1 dx \frac{1}{[(k + zP_x)^2 - z^2 P_x^2 - (1-z)\mu^2]^3}$$

 $= -e^3 \int_0^1 2z\,dz \int_0^1 dx \int \frac{d^4k}{(2\pi)^4} \frac{4\,\bar{u}'\gamma^\mu u\,(p\cdot p')}{(k^2 \underbrace{- z^2 p_x^2 - (1-z)\mu^2}_{-\Delta})}$

$$= -4e^3\,\bar{u}'\gamma^\mu u\,(p\cdot p') \int_0^1 dz\,2z \int_0^1 dx\,\frac{1}{(2\pi)^4}\,\frac{-i\pi^2}{2\left(z^2 p_x^2 + \underbrace{(1-z)}_{\approx 1}\mu^2\right)}$$

Kada $\mu^2 \to 0$ $\int dz$ je divergentan

$$\frac{1}{p_x^2}\int_0^1 \frac{dz^2}{z^2 + \underbrace{(1-z)}_{\approx 1}\underbrace{\frac{\mu^2}{p_x^2}}_{\text{ovo je malo}}} = \frac{1}{p_x^2}\ln\frac{p_x^2+\mu^2}{\mu^2} + \underbrace{\cdots}_{\text{ide u 0 kada } \mu\to}$$

$$p_x^2 = m^2 - x(1-x)q^2$$

V. dij $= -4e^3\,\bar{u}(p')\gamma^\mu u(p)\,(pp')\,\frac{-i}{32\pi^2}\underbrace{\int_0^1 dx\,\frac{1}{m^2 - x(1-x)q^2}\ln\frac{m^2 - x(1-x)q^2}{\mu^2}}_{I}$

$$I \underset{|q^2|\gg m^2}{\sim} \ln\left(-\frac{q^2}{\mu^2}\right)\int_0^1 dx\,\frac{1}{m^2 - x(1-x)q^2}$$

$$= \ln\left(-\frac{q^2}{\mu^2}\right)\frac{-2}{q^2}\ln\left(-\frac{q^2}{m^2}\right)$$

Dijagram $= \frac{ie^3}{8\pi^2}\,\bar{u}(p')\gamma^\mu u(p)\underbrace{\ln\left(-\frac{q^2}{\mu^2}\right)\ln\left(-\frac{q^2}{m^2}\right)}_{\text{double log}}$

 + 

$$= i\mathcal{M} = -ie\,\bar{u}'\not{A}(\vec{q})\,u(p)\left(1-\frac{e^2}{8\pi^2}\ln\left(-\frac{q^2}{\mu^2}\right)\ln\left(-\frac{q^2}{m^2}\right)\right)$$

$$\boxed{d\sigma_{Coul} = (d\sigma)^{0}_{Coul}\left(1-\overset{=\frac{e^2}{4\pi}}{\frac{\alpha}{\pi}}\ln\left(-\frac{q^2}{\mu^2}\right)\ln\left(-\frac{q^2}{m^2}\right)\right)}$$

$\mu \to 0$ $d\sigma_{Coul}$ je IC divergentan

Ukupno:

$$(d\sigma)_{Coul} + (d\sigma)_B = (d\sigma)_0\left[1-\frac{\alpha}{\pi}\ln\left(-\frac{q^2}{m^2}\right)\ln\left(-\frac{q^2}{k_{max}^2}\right)\right]$$

je konačan u $O(\alpha)$ redu i konačan je za konačnu rezoluciju instrumenta

Poništavanje IC divergencija zbog realnih i virtuelnih fotona se dešava u svakom redu teorije perturbacije