Унутрашње симетрије

Елементарне честице

1) лептони $\begin{pmatrix} e^- \\ \nu_e \end{pmatrix}, \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix}, \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}$

2) хадрони - састављени од кваркова < мезони - бозони $q\bar{q}$ / бариони - фермиони qqq
   кваркови: $\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} s \\ c \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}$

3) преносиоци интеракција (гравитон, фотон, глуони, $W^\pm$, Z)
   грав. ем. јака слаба

Група симетрије је $P \otimes G$; P-Поенкареова група, G - унутрашња група сим.
Генератори P комутирају са генераторима G → мултиплети са истом масом и спином.

Изоспинска SU(2) група

Јаке интеракције су приближно независне од наелектрисања нуклеона $\begin{pmatrix} p \\ n \end{pmatrix}$ - изоспински дублет, симетрија је приближна јер $m_p \approx m_n$ (нису исте).

SU(2) - алгебра $I_i = \frac{1}{2}\sigma_i$, σ_i - Паулијеве матрице, $\sigma_1 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}$, $\sigma_2 = \begin{pmatrix} & -i \\ i & \end{pmatrix}$, $\sigma_3 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$

$[I_i, I_j] = i\varepsilon_{ijk} I_k$; Казимиров оператор је $I^2 = \sum_i I_i^2$

елементи SU(2) групе $U = e^{-i\alpha_i \sigma_i \frac{1}{2}}$; IR - репрезентације $D^{(I)}$ - $2I+1$ -дим.

$\{|I, I_3\rangle\}$ базис у $D^{(I)}$, $I(I+1)$ - св. вредности од I^2

$I^2 |I, I_3\rangle = I(I+1) |I, I_3\rangle$; $I_3 |I, I_3\rangle = I_3 |I, I_3\rangle$

оператори подизања и спуштања $I_\pm = I_1 \pm iI_2$; $I_\pm |I, I_3\rangle = \sqrt{I(I+1) - I_3(I_3 \pm 1)}\,|I, I_3 \pm 1\rangle$

2.1. Електрон у атому H_2 се налази у орбитном стању $|2,-1\rangle$ и спинском стању $|\frac{1}{2},\frac{1}{2}\rangle$. Које вредности ће се добити када се мери J^2 електрона и са којом вероватноћом?

Стање електрона је $|2\ -1\rangle|\frac{1}{2}\ \frac{1}{2}\rangle$

$|2,-1\rangle$ је из $D^{(2)}$ - 5d репрезентације

$|\frac{1}{2},\frac{1}{2}\rangle$ је из $D^{(\frac{1}{2})}$ - 2d репрезентације

$$D^{(2)} \otimes D^{(\frac{1}{2})} = D^{(\frac{5}{2})} \oplus D^{(\frac{3}{2})}$$

Вектори из $D^{(\frac{5}{2})}$ и $D^{(\frac{3}{2})}$ репрезентација изражени преко вектора из $D^{(2)} \otimes D^{(\frac{1}{2})}$

$|\frac{5}{2},\frac{5}{2}\rangle = |2\ 2\rangle|\frac{1}{2}\ \frac{1}{2}\rangle$ ← стање максималне тежине

$J_\pm|J,J_3\rangle = \sqrt{J(J+1) \pm J_3(J_3 \pm 1)}\,|J,J_3 \pm 1\rangle$ $J_\pm$ - оператори подизања и спуштања

$$J_-|\tfrac{5}{2},\tfrac{5}{2}\rangle = \sqrt{\tfrac{5}{2}\cdot\tfrac{7}{2} - \tfrac{5}{2}\cdot\tfrac{3}{2}}\,|\tfrac{5}{2},\tfrac{3}{2}\rangle = \sqrt{5}\,|\tfrac{5}{2},\tfrac{3}{2}\rangle = (J_- \otimes 1 + 1 \otimes J_-)|2,2\rangle \otimes |\tfrac{1}{2}\ \tfrac{1}{2}\rangle$$

$$= \sqrt{2\cdot 3 - 2\cdot 1}\,|2,1\rangle|\tfrac{1}{2}\ \tfrac{1}{2}\rangle + |2,2\rangle\sqrt{\tfrac{1}{2}\cdot\tfrac{3}{2} - \tfrac{1}{2}\cdot(-\tfrac{1}{2})}\,|\tfrac{1}{2}\ -\tfrac{1}{2}\rangle = 2|2,1\rangle|\tfrac{1}{2}\ \tfrac{1}{2}\rangle + |2,2\rangle.|\tfrac{1}{2},-\tfrac{1}{2}\rangle$$

$$\Rightarrow |\tfrac{5}{2},\tfrac{3}{2}\rangle = \tfrac{2}{\sqrt{5}}|2,1\rangle|\tfrac{1}{2}\ \tfrac{1}{2}\rangle + \tfrac{1}{\sqrt{5}}|2,2\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle$$

$$J_-|\tfrac{5}{2},\tfrac{3}{2}\rangle = \sqrt{8}\,|\tfrac{5}{2}\ \tfrac{1}{2}\rangle = \tfrac{2}{\sqrt{5}}\left(\sqrt{6}|2,0\rangle|\tfrac{1}{2},\tfrac{1}{2}\rangle + |2,1\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle\right) + \tfrac{1}{\sqrt{5}}\cdot 2|2,1\rangle|\tfrac{1}{2}\ -\tfrac{1}{2}\rangle$$

$$\Rightarrow |\tfrac{5}{2},\tfrac{1}{2}\rangle = \sqrt{\tfrac{3}{5}}|2,0\rangle|\tfrac{1}{2},\tfrac{1}{2}\rangle + \sqrt{\tfrac{2}{5}}|2\,1\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle$$

$$J_-|\tfrac{5}{2},\tfrac{1}{2}\rangle = 3|\tfrac{5}{2},-\tfrac{1}{2}\rangle = \sqrt{\tfrac{3}{5}}\left(\sqrt{6}|2,-1\rangle|\tfrac{1}{2}\ \tfrac{1}{2}\rangle + |2,0\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle\right) + \sqrt{\tfrac{2}{5}}\sqrt{6}|2,0\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle$$

$$\Rightarrow |\tfrac{5}{2},-\tfrac{1}{2}\rangle = \sqrt{\tfrac{2}{5}}|2,-1\rangle|\tfrac{1}{2}\ \tfrac{1}{2}\rangle + \sqrt{\tfrac{3}{5}}|2,0\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle$$

$$J_-|\tfrac{5}{2},-\tfrac{1}{2}\rangle = \sqrt{8}\,|\tfrac{5}{2},-\tfrac{3}{2}\rangle = \sqrt{\tfrac{2}{5}}\left(2|2,-2\rangle|\tfrac{1}{2}\ \tfrac{1}{2}\rangle + |2,-1\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle\right) + \sqrt{\tfrac{3}{5}}\sqrt{6}|2,-1\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle$$

$$\Rightarrow |\tfrac{5}{2},-\tfrac{3}{2}\rangle = \sqrt{\tfrac{1}{5}}|2,-2\rangle|\tfrac{1}{2},\tfrac{1}{2}\rangle + \tfrac{2}{\sqrt{5}}|2,-1\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle$$

$$J_-|\tfrac{5}{2},-\tfrac{3}{2}\rangle = \sqrt{5}\,|\tfrac{5}{2},-\tfrac{5}{2}\rangle = \tfrac{1}{\sqrt{5}}|2,-2\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle + \tfrac{4}{\sqrt{5}}|2,-2\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle$$

$\Rightarrow |\frac{5}{2},-\frac{5}{2}\rangle = |2,-2\rangle|\frac{1}{2},-\frac{1}{2}\rangle$ ← стање најмање тежине

$|\frac{3}{2}\ \frac{3}{2}\rangle = a|2,1\rangle|\frac{1}{2}\ \frac{1}{2}\rangle + b|2,2\rangle|\frac{1}{2},-\frac{1}{2}\rangle$ ово стање је ортогонално на $|\frac{5}{2},\frac{3}{2}\rangle$

$\langle\frac{5}{2},\frac{3}{2}|\frac{3}{2},\frac{3}{2}\rangle = a\frac{2}{\sqrt{5}} + b\frac{1}{\sqrt{5}} = 0 \Rightarrow b = -2a$; $a^2 + b^2 = 1 = 5a^2 \Rightarrow a = \frac{1}{\sqrt{5}}$; $b = -\frac{2}{\sqrt{5}}$

$$\Rightarrow \left|\tfrac{3}{2},\tfrac{3}{2}\right\rangle = \frac{1}{\sqrt5}\left|2,1\right\rangle\left|\tfrac12\,\tfrac12\right\rangle - \frac{2}{\sqrt5}\left|2,2\right\rangle\left|\tfrac12,-\tfrac12\right\rangle$$

$$J_-\left|\tfrac32,\tfrac32\right\rangle = \sqrt3\left|\tfrac32,\tfrac12\right\rangle = \frac{1}{\sqrt5}\left(\sqrt6\left|2,0\right\rangle\left|\tfrac12,\tfrac12\right\rangle + \left|2,1\right\rangle\left|\tfrac12,-\tfrac12\right\rangle\right) - \frac{2}{\sqrt5}\cdot 2\left|2,1\right\rangle\left|\tfrac12,-\tfrac12\right\rangle$$

$$\Rightarrow \left|\tfrac32,\tfrac12\right\rangle = \sqrt{\tfrac25}\left|2,0\right\rangle\left|\tfrac12,\tfrac12\right\rangle - \sqrt{\tfrac35}\left|2,1\right\rangle\left|\tfrac12,-\tfrac12\right\rangle$$

$$J_-\left|\tfrac32,\tfrac12\right\rangle = 2\left|\tfrac32,-\tfrac12\right\rangle = \sqrt{\tfrac25}\left(\sqrt6\left|2,-1\right\rangle\left|\tfrac12,\tfrac12\right\rangle + \left|2,0\right\rangle\left|\tfrac12,-\tfrac12\right\rangle\right) - \sqrt{\tfrac35}\sqrt6\left|2,0\right\rangle\left|\tfrac12,-\tfrac12\right\rangle$$

$$\Rightarrow \left|\tfrac32\,-\tfrac12\right\rangle = \sqrt{\tfrac35}\left|2,-1\right\rangle\left|\tfrac12\,\tfrac12\right\rangle - \sqrt{\tfrac25}\left|2,0\right\rangle\left|\tfrac12,-\tfrac12\right\rangle$$

$$J_-\left|\tfrac32,-\tfrac12\right\rangle = \sqrt3\left|\tfrac32,-\tfrac32\right\rangle = \sqrt{\tfrac35}\left(2\left|2,-2\right\rangle\left|\tfrac12\,\tfrac12\right\rangle + \left|2,-1\right\rangle\left|\tfrac12,-\tfrac12\right\rangle\right) - \sqrt{\tfrac25}\sqrt6\left|2,-1\right\rangle\left|\tfrac12,-\tfrac12\right\rangle$$

$$\Rightarrow \left|\tfrac32,-\tfrac32\right\rangle = \frac{2}{\sqrt5}\left|2,-2\right\rangle\left|\tfrac12,\tfrac12\right\rangle - \frac{1}{\sqrt5}\left|2,-1\right\rangle\left|\tfrac12,-\tfrac12\right\rangle$$

$$\langle J^2\rangle = \left\langle 2,-1;\tfrac12\,\tfrac12\right| J^2 \left|2,-1;\tfrac12,\tfrac12\right\rangle = \sum_{J,J_3}\left\langle 2,-1;\tfrac12\,\tfrac12\right|J^2\left|J,J_3\right\rangle\left\langle J,J_3\middle|2,-1;\tfrac12,\tfrac12\right\rangle$$

$$= \sum_{JJ_3} J(J+1)\Big|\underbrace{\left\langle 2,-1;\tfrac12,\tfrac12\middle|J,J_3\right\rangle}_{\text{CG -коефицијент}}\Big|^2$$

Ненулти CG коефицијенти су:

$$\left\langle 2,-1;\tfrac12\tfrac12\middle|\tfrac52,-\tfrac12\right\rangle = \sqrt{\tfrac25} \quad \text{и} \quad \left\langle 2,-1;\tfrac12,\tfrac12\middle|\tfrac32,-\tfrac12\right\rangle = \sqrt{\tfrac35}$$

$$\langle J^2\rangle_{|\psi\rangle} = \frac52\cdot\frac72\cdot\left(\frac25\right) + \frac32\cdot\frac52\cdot\left(\frac35\right)$$

Оператор J^2 има вредност $\frac{35}{4}$ са вероватноћом $\frac25$ и $\frac{15}{4}$ са вероват. $\frac35$

2.2. Користећи операторе $I_\pm$, одредити CG коефицијенте у разлагању директног производа иредуцибилних репрезентација $I=\frac12$ и $I=1$. Упоредити добијене вредности са табличним.

$$1\otimes\tfrac12 = \tfrac32\oplus\tfrac12$$

dim: $3\cdot 2 = 4+2$

У репрезентацији 1 базис: $|1,1\rangle,\ |1,0\rangle,\ |1,-1\rangle$

$$I_-|1,1\rangle = \sqrt2\,|1,0\rangle\ ;\ I_-|1,0\rangle = \sqrt2\,|1,-1\rangle;\ I_-|1,-1\rangle = 0$$

$$I_-\left|\tfrac12,\tfrac12\right\rangle = \left|\tfrac12,-\tfrac12\right\rangle$$

У репрезентацији $\frac{3}{2}$ базис:

$$|\tfrac{3}{2},\tfrac{3}{2}\rangle = |1,1\rangle|\tfrac{1}{2},\tfrac{1}{2}\rangle$$

$$I_-|\tfrac{3}{2}\ \tfrac{3}{2}\rangle = \sqrt{3}\,|\tfrac{3}{2}\ \tfrac{1}{2}\rangle = \sqrt{2}\,|1,0\rangle|\tfrac{1}{2},\tfrac{1}{2}\rangle + |1,1\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle$$

$$\Rightarrow |\tfrac{3}{2},\tfrac{1}{2}\rangle = \sqrt{\tfrac{2}{3}}\,|1,0\rangle|\tfrac{1}{2},\tfrac{1}{2}\rangle + \frac{1}{\sqrt{3}}|1,1\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle$$

$$I_-|\tfrac{3}{2},\tfrac{1}{2}\rangle = 2\,|\tfrac{3}{2},-\tfrac{1}{2}\rangle = \sqrt{\tfrac{2}{3}}\left(\sqrt{2}\,|1,-1\rangle|\tfrac{1}{2},\tfrac{1}{2}\rangle + |1,0\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle\right) + \sqrt{\tfrac{2}{3}}\,|1,0\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle$$

$$\Rightarrow |\tfrac{3}{2},-\tfrac{1}{2}\rangle = \frac{1}{\sqrt{3}}|1,-1\rangle|\tfrac{1}{2},\tfrac{1}{2}\rangle + \sqrt{\tfrac{2}{3}}\,|1,0\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle$$

$$I_-|\tfrac{3}{2},-\tfrac{1}{2}\rangle = \sqrt{3}\,|\tfrac{3}{2},-\tfrac{3}{2}\rangle = \frac{1}{\sqrt{3}}|1,-1\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle + \frac{2}{\sqrt{3}}|1,-1\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle$$

$$\Rightarrow |\tfrac{3}{2},-\tfrac{3}{2}\rangle = |1,-1\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle$$

$$|\tfrac{1}{2}\ \tfrac{1}{2}\rangle = a\,|1,0\rangle|\tfrac{1}{2}\ \tfrac{1}{2}\rangle + b\,|1,1\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle$$

$$\langle\tfrac{1}{2},\tfrac{1}{2}|\tfrac{3}{2},\tfrac{1}{2}\rangle = 0 = a\cdot\sqrt{\tfrac{2}{3}} + b\,\frac{1}{\sqrt{3}} = 0 \Rightarrow \quad b = -a\sqrt{2}\,;\ a^2+b^2=1 \Rightarrow a = -\frac{1}{\sqrt{3}}\,;\ b = +\sqrt{\tfrac{2}{3}}$$

$$|\tfrac{1}{2},\tfrac{1}{2}\rangle = -\frac{1}{\sqrt{3}}|1,0\rangle|\tfrac{1}{2},\tfrac{1}{2}\rangle + \sqrt{\tfrac{2}{3}}\,|1,1\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle$$

$$I_-|\tfrac{1}{2},\tfrac{1}{2}\rangle = |\tfrac{1}{2},-\tfrac{1}{2}\rangle = -\sqrt{\tfrac{2}{3}}\,|1,-1\rangle|\tfrac{1}{2},\tfrac{1}{2}\rangle - \frac{1}{\sqrt{3}}|1,0\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle + \frac{2}{\sqrt{3}}|1,0\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle$$

$$|\tfrac{1}{2},-\tfrac{1}{2}\rangle = -\sqrt{\tfrac{2}{3}}\,|1,-1\rangle|\tfrac{1}{2},\tfrac{1}{2}\rangle + \frac{1}{\sqrt{3}}|1,0\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle$$

CG коефицијенти: (ненулти)

$\langle 1,1;\tfrac{1}{2},\tfrac{1}{2}|\tfrac{3}{2},\tfrac{3}{2}\rangle = 1$; $\langle 1,0;\tfrac{1}{2}\tfrac{1}{2}|\tfrac{3}{2},\tfrac{1}{2}\rangle = \sqrt{\tfrac{2}{3}}$; $\langle 1,1;\tfrac{1}{2},-\tfrac{1}{2}|\tfrac{3}{2},\tfrac{1}{2}\rangle = \frac{1}{\sqrt{3}}$

$\langle 1,-1;\tfrac{1}{2},\tfrac{1}{2}|\tfrac{3}{2},-\tfrac{1}{2}\rangle = \frac{1}{\sqrt{3}}$; $\langle 1,0;\tfrac{1}{2},-\tfrac{1}{2}|\tfrac{3}{2},-\tfrac{1}{2}\rangle = \sqrt{\tfrac{2}{3}}$; $\langle 1,-1;\tfrac{1}{2},-\tfrac{1}{2}|\tfrac{3}{2},-\tfrac{3}{2}\rangle = 1$

$\langle 1,0;\tfrac{1}{2},\tfrac{1}{2}|\tfrac{1}{2},\tfrac{1}{2}\rangle = -\frac{1}{\sqrt{3}}$; $\langle 1,1\ \tfrac{1}{2},-\tfrac{1}{2}\rangle = \sqrt{\tfrac{2}{3}}$; $\langle 1,-1;\tfrac{1}{2},\tfrac{1}{2}|\tfrac{1}{2},-\tfrac{1}{2}\rangle = -\sqrt{\tfrac{2}{3}}$

$\langle 1,0;\tfrac{1}{2},-\tfrac{1}{2}|\tfrac{1}{2},-\tfrac{1}{2}\rangle = \frac{1}{\sqrt{3}}$

Користећи резултате задатка 2.2. одредити:

а) однос ширина распада $\Delta^+ \to p+\pi^0$ и $\Delta^+ \to n+\pi^+$.

б) однос ефикасних пресека за расејања:

$$p+\pi^+ \to p+\pi^+ \; ; \; p+\pi^- \to p+\pi^- \; ; \; p+\pi^- \to n+\pi^0 .$$

Пиони π^+, π^0 и π^- представљају триплет изоспински

$$\pi^+ = |1,1\rangle \; ; \; \pi^0 = |1,0\rangle \; ; \; \pi^- = |1,-1\rangle$$

Протон и неутрон p, n, представљају изоспински дублет

$$p = |\tfrac{1}{2}, \tfrac{1}{2}\rangle \; ; \; n = |\tfrac{1}{2}, -\tfrac{1}{2}\rangle$$

$ab \to cd$ $$\langle cd | S | ab\rangle = \sum_{I,I'} \langle I_c, I_d ; I_{c3}, I_{d3} | I I_3\rangle \underbrace{\langle I, I_3 | S | I', I_3'\rangle}_{\delta_{II'}\delta_{I_3 I_3'} A_I} \langle I' I_3' | I_a, I_b ; I_{a3}, I_{b3}\rangle$$

$$= \sum_I A_I \langle I_c, I_d ; I_{c3}, I_{d3} | I I_3\rangle \langle I I_3 | I_a, I_b ; I_{a3}, I_{b3}\rangle$$

$p+\pi^+ \to p+\pi^+$ $\quad |p\pi^+\rangle = |\tfrac{1}{2}, 1; \tfrac{1}{2}, 1\rangle = |\tfrac{1}{2}, 1; \tfrac{3}{2} \tfrac{3}{2}\rangle$

$$\langle p\pi^+ | S | p\pi^+\rangle = A_{3/2}$$

$p+\pi^- \to p+\pi^-$ $\quad |p\pi^-\rangle = |\tfrac{1}{2}, 1; \tfrac{1}{2}, -1\rangle$

$$\langle p\pi^- | S | p\pi^-\rangle = A_{1/2} \underbrace{|\langle p\pi^- | \tfrac{1}{2}, -\tfrac{1}{2}\rangle|^2}_{2/3} + A_{3/2} \underbrace{|\langle p\pi^- | \tfrac{3}{2}, -\tfrac{1}{2}\rangle|^2}_{1/3} = \frac{2}{3} A_{1/2} + \frac{1}{3} A_{3/2}$$

$p+\pi^- \to n+\pi^0$ $\quad |p\pi^-\rangle = |\tfrac{1}{2}, 1; \tfrac{1}{2} -1\rangle \; ; \; |n\pi^0\rangle = |\tfrac{1}{2}, 1; -\tfrac{1}{2}, 0\rangle$

$$\langle n\pi^0 | S | p\pi^-\rangle = A_{1/2} \underbrace{\langle n\pi^0 | \tfrac{1}{2}, -\tfrac{1}{2}\rangle}_{+\sqrt{\frac{1}{3}}} \underbrace{\langle \tfrac{1}{2}, -\tfrac{1}{2} | p\pi^-\rangle}_{-\sqrt{\frac{2}{3}}} + A_{3/2} \underbrace{\langle n\pi^0 | \tfrac{3}{2}, -\tfrac{1}{2}\rangle}_{\sqrt{\frac{2}{3}}} \underbrace{\langle \tfrac{3}{3}, -\tfrac{1}{2} | p\pi^-\rangle}_{\sqrt{\frac{1}{3}}}$$

$$\sigma_a = \sigma(p+\pi^+ \to p+\pi^+) \sim |\langle p\pi^+ | S | p\pi^+\rangle|^2 = A_{3/2}^2$$

$$\sigma_b = \sigma(p+\pi^- \to p+\pi^-) \sim |\langle p\pi^- | S | p\pi^-\rangle|^2 = \frac{1}{9}(2A_{1/2} + A_{3/2})^2$$

$$\sigma_c = \sigma(p+\pi^- \to n+\pi^0) \sim |\langle n\pi^0 | S | p\pi^-\rangle|^2 = \left| -\frac{\sqrt{2}}{3} A_{1/2} + \frac{\sqrt{2}}{3} A_{3/2} \right|^2$$

експерим. $\boxed{A_{3/2} \gg A_{1/2}}$ $\quad \sigma_a = A_{3/2}^2 \; ; \; \sigma_b \approx \frac{1}{9} A_{3/2}^2 \; ; \; \sigma_c \not\approx \frac{2}{9} A_{3/2}^2$

$$\sigma_a : \sigma_b : \sigma_c = A_{3/2}^2 : \frac{1}{9} A_{3/2}^2 : \frac{2}{9} A_{3/2}^2 = 9 : 1 : 2$$

експериментално, лакше се мери $\quad \dfrac{\sigma_{tot}(\pi^+ + p)}{\sigma_{tot}(\pi^- + p)} = \dfrac{9}{3} = 3$

Са графика се види да је $\quad \dfrac{\sigma_{tot}(\pi^+ + p)}{\sigma_{tot}(\pi^- + p)} \approx \dfrac{195}{65} = 3$

б) однос ефикасних пресека за расејања $p+p \to d+\pi^+$; $p+n \to d+\pi^0$, где је

Ж d ознака за деутерон, који је изоспински синглет.

$$a+b \longrightarrow c+d$$

$$\langle cd | S | ab \rangle = \sum_{I I_3} \langle cd | I, I_3 \rangle A_I \langle I, I_3 | ab \rangle$$

p, n изоспински дублет π^+, π^0, π^- изоспински триплет

$p = |\tfrac{1}{2}\ \tfrac{1}{2}\rangle$ $n = |\tfrac{1}{2}\ -\tfrac{1}{2}\rangle$ $\pi^+ = |1,1\rangle$; $\pi^0 = |1,0\rangle$; $\pi^- = |1,-1\rangle$

$|d\pi^+\rangle = |1,1\rangle$; $|pp\rangle = |\tfrac{1}{2}, \tfrac{1}{2} ; \tfrac{1}{2}, \tfrac{1}{2}\rangle$

$$\langle d\pi^+ | S | pp \rangle = A_1 \langle 1,1 | \tfrac{1}{2}\ \tfrac{1}{2} ; \tfrac{1}{2}, \tfrac{1}{2}\rangle = A_1$$

чисто стање!

$|d\pi^0\rangle = |1,0\rangle$; $|pn\rangle = |\tfrac{1}{2}, \tfrac{1}{2} ; \tfrac{1}{2}, -\tfrac{1}{2}\rangle$

$$\langle d\pi^0 | S | pn \rangle = A_1 \langle 1,0 | \tfrac{1}{2}, \tfrac{1}{2} ; \tfrac{1}{2}, -\tfrac{1}{2}\rangle = A_1 \frac{1}{\sqrt{2}}$$

$$\frac{\sigma(p+p \to d\pi^+)}{\sigma(pn \to d+\pi^0)} = \frac{A_1^2}{A_1^2 \frac{1}{2}} = 2$$

а) $\Delta^+ \to p+\pi^0$ $\Delta^+ \to n+\pi^+$

$a \to cd$ $\langle cd | S | a \rangle = \sum_I \langle cd | I, I_3 \rangle \langle I, I_3 | S | I_a, I_{a3} \rangle$

$$= \langle cd | I_a, I_{a3} \rangle A_{I_a}$$

$\Gamma(a \to cd) \sim |\langle cd | a \rangle|^2$

Δ резонанце квартет $SU(2)$ група репрез. $D^{(3/2)}$

$\Delta^{++} = |\tfrac{3}{2}, \tfrac{3}{2}\rangle$ $\boxed{\Delta^+ = |\tfrac{3}{2}, \tfrac{1}{2}\rangle}$ $\Delta^0 = |\tfrac{3}{2}, -\tfrac{1}{2}\rangle$ $\Delta^- = |\tfrac{3}{2}, -\tfrac{3}{2}\rangle$

(1×½) $\Delta^+ \to \overset{1/2}{p} + \overset{1}{\pi^0}$

$|p\pi^0\rangle = |\tfrac{1}{2}, \tfrac{1}{2}\rangle |1,0\rangle$ $|n\pi^+\rangle = |\tfrac{1}{2}, -\tfrac{1}{2}\rangle |1,1\rangle$

$\langle p\pi^0 | \Delta^+ \rangle = -\sqrt{\frac{2}{3}}$; $\langle n\pi^+ | \Delta^+ \rangle = \frac{1}{\sqrt{3}}$

$$\frac{\sigma(\Delta^+ \to p\pi^0)}{\sigma(\Delta^+ \to n\pi^+)} = \frac{\frac{2}{3}}{\frac{1}{3}} = 2$$

36. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for −8/15 read $-\sqrt{8/15}$.

Notation:

		J	J	...
		M	M	...
m_1	m_2	Coefficients		
m_1	m_2			
.	.			
.	.			

1/2×1/2

		1
		+1
+1/2	+1/2	1

		1	0
		0	0
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2

		1
		−1
−1/2	−1/2	1

1×1/2

		3/2
		+3/2
+1	+1/2	1

		3/2	1/2
		+1/2	+1/2
+1	−1/2	1/3	2/3
0	+1/2	2/3	−1/3

		3/2	1/2
		−1/2	−1/2
0	−1/2	2/3	1/3
−1	+1/2	1/3	−2/3

		3/2
		−3/2
−1	−1/2	1

2×1

		3
		+3
+2	+1	1

		3	2
		+2	+2
+2	0	1/3	2/3
+1	+1	2/3	−1/3

		3	2	1
		+1	+1	+1
+2	−1	1/15	1/3	3/5
+1	0	8/15	1/6	−3/10
0	+1	2/5	−1/2	1/10

		3	2	1
		0	0	0
+1	−1	1/5	1/2	3/10
0	0	3/5	0	−2/5
−1	+1	1/5	−1/2	3/10

		3	2	1
		−1	−1	−1
0	−1	2/5	1/2	1/10
−1	0	8/15	−1/6	−3/10
−2	+1	1/15	−1/3	3/5

		3	2
		−2	−2
−1	−1	2/3	1/3
−2	0	1/3	−2/3

		3
		−3
−2	−1	1

1×1

		2
		+2
+1	+1	1

		2	1
		+1	+1
+1	0	1/2	1/2
0	+1	1/2	−1/2

		2	1	0
		0	0	0
+1	−1	1/6	1/2	1/3
0	0	2/3	0	−1/3
−1	+1	1/6	−1/2	1/3

		2	1
		−1	−1
0	−1	1/2	1/2
−1	0	1/2	−1/2

		2
		−2
−1	−1	1

$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

2×1/2

		5/2
		+5/2
+2	+1/2	1

		5/2	3/2
		+3/2	+3/2
+2	−1/2	1/5	4/5
+1	+1/2	4/5	−1/5

		5/2	3/2
		+1/2	+1/2
+1	−1/2	2/5	3/5
0	+1/2	3/5	−2/5

		5/2	3/2
		−1/2	−1/2
0	−1/2	3/5	2/5
−1	+1/2	2/5	−3/5

		5/2	3/2
		−3/2	−3/2
−1	−1/2	4/5	1/5
−2	+1/2	1/5	−4/5

		5/2
		−5/2
−2	−1/2	1

3/2×1/2

		2
		+2
+3/2	+1/2	1

		2	1
		+1	+1
+3/2	−1/2	1/4	3/4
+1/2	+1/2	3/4	−1/4

		2	1
		0	0
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2

		2	1
		−1	−1
−1/2	−1/2	3/4	1/4
−3/2	+1/2	1/4	−3/4

		2
		−2
−3/2	−1/2	1

3/2×1

		5/2
		+5/2
+3/2	+1	1

		5/2	3/2
		+3/2	+3/2
+3/2	0	2/5	3/5
+1/2	+1	3/5	−2/5

		5/2	3/2	1/2
		+1/2	+1/2	+1/2
+3/2	−1	1/10	2/5	1/2
+1/2	0	3/5	1/15	−1/3
−1/2	+1	3/10	−8/15	1/6

		5/2	3/2	1/2
		−1/2	−1/2	−1/2
+1/2	−1	3/10	8/15	1/6
−1/2	0	3/5	−1/15	−1/3
−3/2	+1	1/10	−2/5	1/2

		5/2	3/2
		−3/2	−3/2
−1/2	−1	3/5	2/5
−3/2	0	2/5	−3/5

		5/2
		−5/2
−3/2	−1	1

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

$$d^{\ell}_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

$$d^{j}_{m',m} = (-1)^{m-m'} d^{j}_{m,m'} = d^{j}_{-m,-m'}$$

2×3/2

		7/2
		+7/2
+2	+3/2	1

		7/2	5/2
		+5/2	+5/2
+2	+1/2	3/7	4/7
+1	+3/2	4/7	−3/7

		7/2	5/2	3/2
		+3/2	+3/2	+3/2
+2	−1/2	1/7	16/35	2/5
+1	+1/2	4/7	1/35	−2/5
0	+3/2	2/7	−18/35	1/5

		7/2	5/2	3/2	1/2
		+1/2	+1/2	+1/2	+1/2
+2	−3/2	1/35	6/35	2/5	2/5
+1	−1/2	12/35	5/14	0	−3/10
0	+1/2	18/35	−3/35	−1/5	1/5
−1	+3/2	4/35	−27/70	2/5	−1/10

		7/2	5/2	3/2	1/2
		−1/2	−1/2	−1/2	−1/2
+1	−3/2	4/35	27/70	2/5	1/10
0	−1/2	18/35	3/35	−1/5	−1/5
−1	+1/2	12/35	−5/14	0	3/10
−2	+3/2	1/35	−6/35	2/5	−2/5

		7/2	5/2	3/2
		−3/2	−3/2	−3/2
0	−3/2	2/7	18/35	1/5
−1	−1/2	4/7	−1/35	−2/5
−2	+1/2	1/7	−16/35	2/5

		7/2	5/2
		−5/2	−5/2
−1	−3/2	4/7	3/7
−2	−1/2	3/7	−4/7

		7/2
		−7/2
−2	−3/2	1

3/2×3/2

		3
		+3
+3/2	+3/2	1

		3	2
		+2	+2
+3/2	+1/2	1/2	1/2
+1/2	+3/2	1/2	−1/2

		3	2	1
		+1	+1	+1
+3/2	−1/2	1/5	1/2	3/10
+1/2	+1/2	3/5	0	−2/5
−1/2	+3/2	1/5	−1/2	3/10

		3	2	1	0
		0	0	0	0
+3/2	−3/2	1/20	1/4	9/20	1/4
+1/2	−1/2	9/20	1/4	−1/20	−1/4
−1/2	+1/2	9/20	−1/4	−1/20	1/4
−3/2	+3/2	1/20	−1/4	9/20	−1/4

		3	2	1
		−1	−1	−1
+1/2	−3/2	1/5	1/2	3/10
−1/2	−1/2	3/5	0	−2/5
−3/2	+1/2	1/5	−1/2	3/10

		3	2
		−2	−2
−1/2	−3/2	1/2	1/2
−3/2	−1/2	1/2	−1/2

		3
		−3
−3/2	−3/2	1

$$d^1_{0,0} = \cos\theta \qquad d^{1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^1_{1,1} = \frac{1+\cos\theta}{2}$$

$$d^{1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2} \qquad d^1_{1,0} = -\frac{\sin\theta}{\sqrt{2}}$$

$$d^1_{1,-1} = \frac{1-\cos\theta}{2}$$

2×2

		4
		+4
+2	+2	1

		4	3
		+3	+3
+2	+1	1/2	1/2
+1	+2	1/2	−1/2

		4	3	2
		+2	+2	+2
+2	0	3/14	1/2	2/7
+1	+1	4/7	0	−3/7
0	+2	3/14	−1/2	2/7

		4	3	2	1
		+1	+1	+1	+1
+2	−1	1/14	3/10	3/7	1/5
+1	0	3/7	1/5	−1/14	−3/10
0	+1	3/7	−1/5	−1/14	3/10
−1	+2	1/14	−3/10	3/7	−1/5

		4	3	2	1	0
		0	0	0	0	0
+2	−2	1/70	1/10	2/7	2/5	1/5
+1	−1	8/35	2/5	1/14	−1/10	−1/5
0	0	18/35	0	−2/7	0	1/5
−1	+1	8/35	−2/5	1/14	1/10	−1/5
−2	+2	1/70	−1/10	2/7	−2/5	1/5

		4	3	2	1
		−1	−1	−1	−1
+1	−2	1/14	3/10	3/7	1/5
0	−1	3/7	1/5	−1/14	−3/10
−1	0	3/7	−1/5	−1/14	3/10
−2	+1	1/14	−3/10	3/7	−1/5

		4	3	2
		−2	−2	−2
0	−2	3/14	1/2	2/7
−1	−1	4/7	0	−3/7
−2	0	3/14	−1/2	2/7

		4	3
		−3	−3
−1	−2	1/2	1/2
−2	−1	1/2	−1/2

		4
		−4
−2	−2	1

$$d^{3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,1/2} = -\sqrt{3}\frac{1+\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-1/2} = \sqrt{3}\frac{1-\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{1/2,1/2} = \frac{3\cos\theta-1}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{1/2,-1/2} = -\frac{3\cos\theta+1}{2}\sin\frac{\theta}{2}$$

$$d^2_{2,2} = \left(\frac{1+\cos\theta}{2}\right)^2$$

$$d^2_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta$$

$$d^2_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta$$

$$d^2_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta$$

$$d^2_{2,-2} = \left(\frac{1-\cos\theta}{2}\right)^2$$

$$d^2_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta-1)$$

$$d^2_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

$$d^2_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta+1)$$

$$d^2_{0,0} = \left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

Figure 36.1: The sign convention is that of Wigner (*Group Theory*, Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and Cohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974).

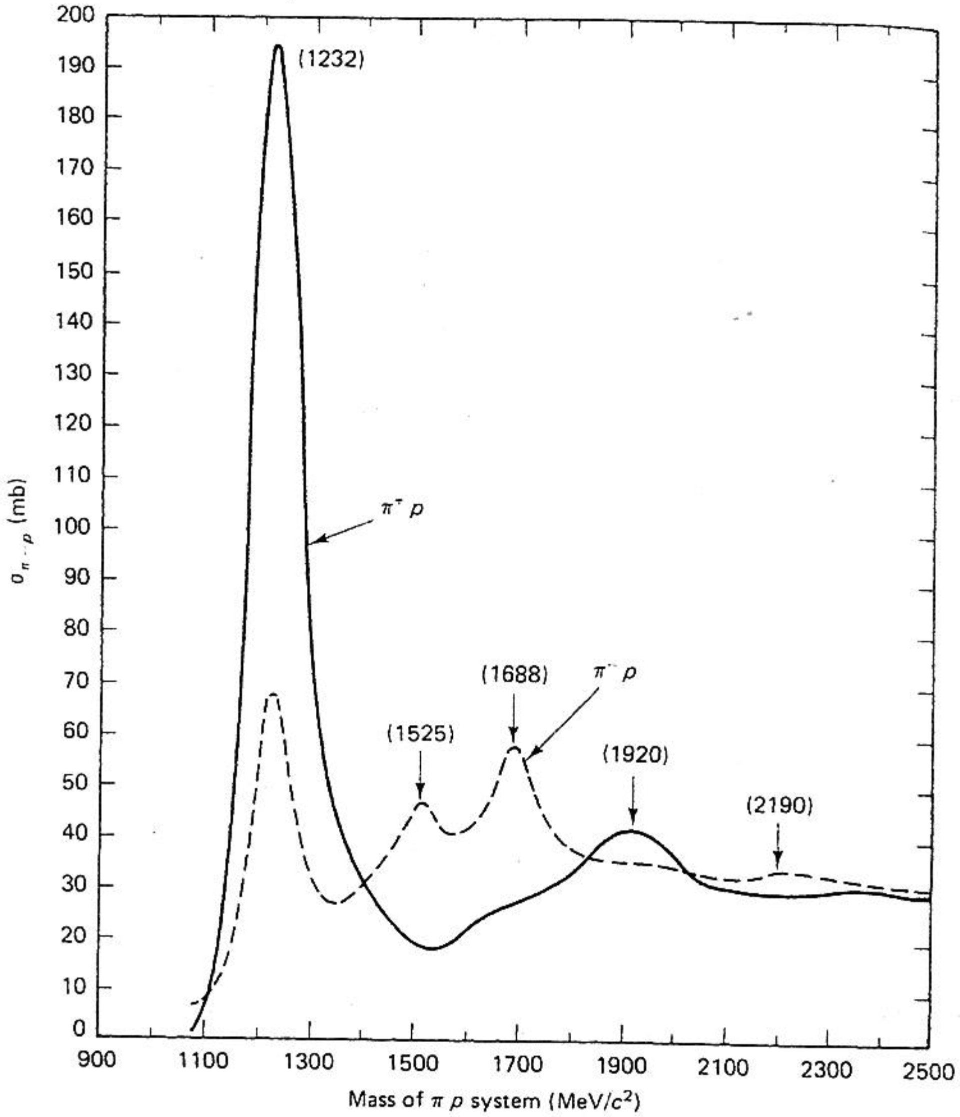


Fig. 4.6 Total cross sections for $\pi^+ p$ (solid line) and $\pi^- p$ (dashed line) scattering. (*Source*: Gasiorowicz, S. (1966) *Elementary Particle Physics*, John Wiley & Sons, New York, p. 294. Reprinted by permission of John Wiley and Sons, Inc.)

occurring particles fall into the fundamental (three-dimensional) representation of $SU(3)$, as the nucleons, and later the K's, the Ξ's, and so on, do for $SU(2)$. This role was reserved for the quarks: u, d, and s together form a three-dimensional representation of $SU(3)$, which breaks down into an isodoublet (u, d) and an isosinglet (s) under $SU(2)$.

Of course, when the charmed quark came along, the flavor symmetry group of the strong interactions expanded once again – this time to $SU(4)$ (some $SU(4)$ supermultiplets are shown in Figure 1.13). But things barely paused there before the arrival of the bottom quark, taking us to $SU(5)$, and finally the top quark, $SU(6)$.

Table 4.4 Quark

Quark flavor

u
d
s
c
b
t

Warning: Tl
speculative

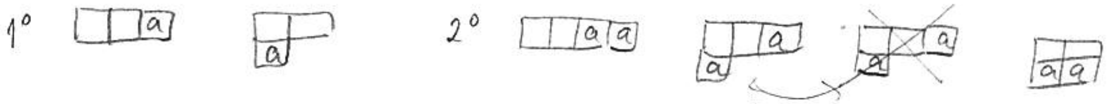
However, th
very 'good' :
most 2 or 3
be expected
splittings w
worse wher
although th
and absolut
Why is is
so poor? Tl
of quark m
direct expe
quarks are
within the
value, in fa
in mesons
inertia of a
tea, and in
effective m
[10] (see T
down qua
masses. B
separated.
all flavors
masses ar
their *bare*
effective

* Indeed, it
was an ex
tions, and
attributab
The fact

Користећи Јунгове шеме, разложити следеће директне производе ир. репрезент. на директне збирове. Резултате проверити рачунајући димензије репрезентација

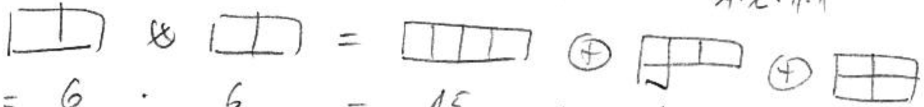
а) SU(3)

$\dim\left(\boxed{3\,|\,4}\right)=\frac{3\cdot\cancel{4}^{2}}{\cancel{2}\cdot 1}=6$



исте са истим словима

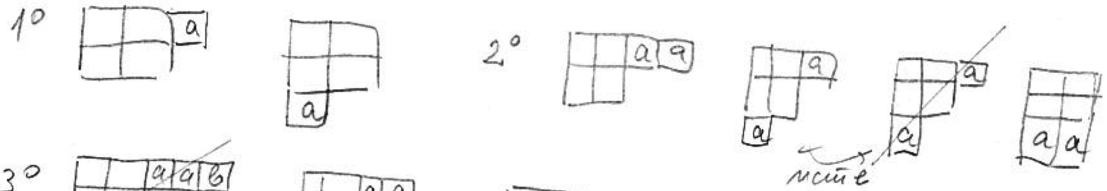
$\dim\left(\boxed{3\,|\,4\,|\,5\,|\,6}\right)=\frac{3\cdot\cancel{4}\cdot5\cdot\cancel{6}}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot1}=15$ $\quad\dim\left(\begin{array}{|c|c|c|}\hline 3&4&5\\\hline 2\\\cline{1-1}\end{array}\right)=\frac{3\cdot\cancel{4}\cdot5\cdot\cancel{2}}{\cancel{4}\cdot\cancel{2}\cdot1\cdot1}=15$ $\quad\dim\begin{array}{|c|c|}\hline 3&4\\\hline 2&3\\\hline\end{array}=\frac{\cancel{3}\cdot4\cdot\cancel{2}\cdot3}{\cancel{3}\cdot\cancel{2}\cdot2}=6$

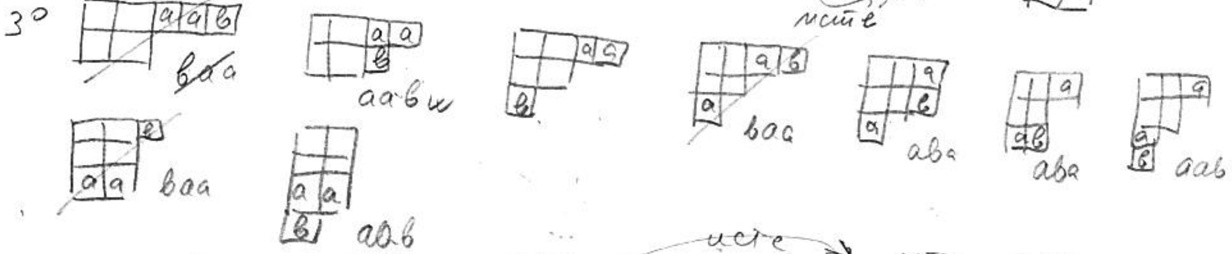


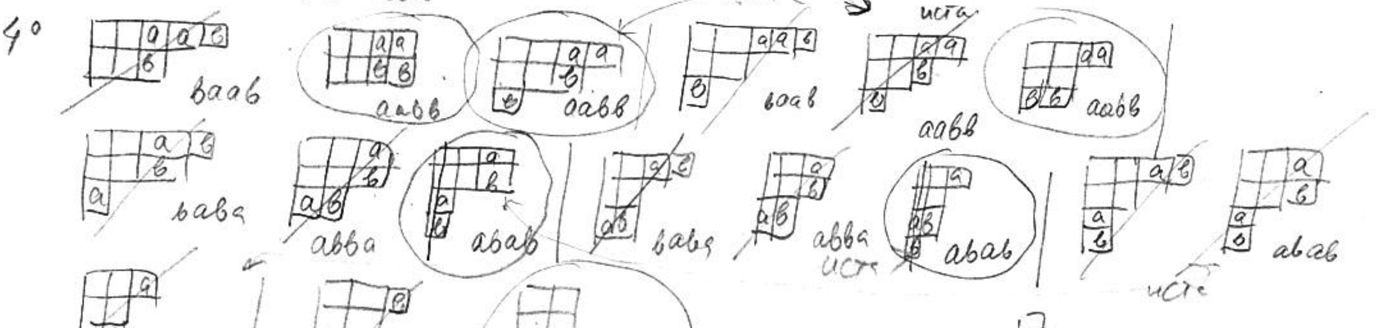
36 = 6 · 6 = 15 + 15 + 6 ✓

б) SU(4)

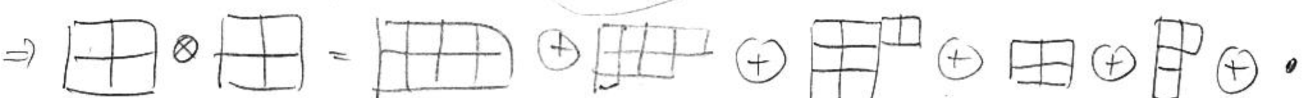
$\dim\begin{array}{|c|c|}\hline 4&5\\\hline 3&4\\\hline\end{array}=\frac{4\cdot5\cdot\cancel{3}\cdot\cancel{4}}{\cancel{3}\cdot\cancel{2}\cdot\cancel{2}\cdot1}=20$







где имамо ▯ то скраћујемо → синглет



20 · 20 = 105 + 175 + 84 + 20 + 15 + 1 ✓

$\dim\left(\begin{array}{llll} 4&5&6&7\\ 3&4&5&6 \end{array}\right) = \frac{}{\cancel{6}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\;\cancel{4}\cdot\cancel{3}\cdot 2\cdot 1} = 15\cdot 7 = 105$

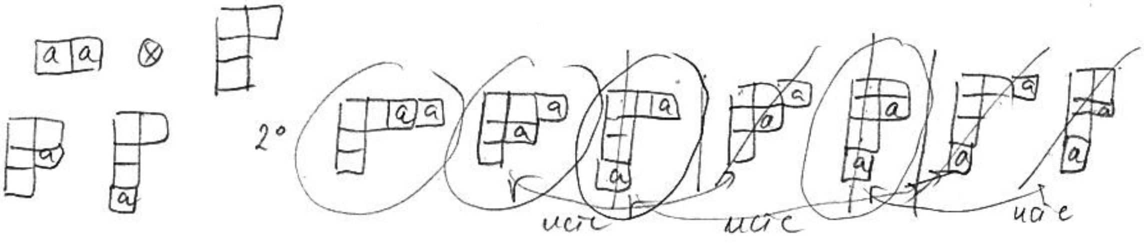
$\dim\left(\begin{array}{llll} 4&5&6&7\\ 3&4&5& \\ 2&&& \end{array}\right) = \frac{\cancel{4}\cdot 5\cdot\cancel{6}\cdot 7\cdot\cancel{3}\cdot\cancel{4}\cdot 5\cdot\cancel{2}}{\cancel{6}\cdot\cancel{4}\cdot\cancel{3}\cdot 1\cdot\cancel{4}\cdot\cancel{2}\cdot 1\cdot 1} = 25\cdot 7 = 175$

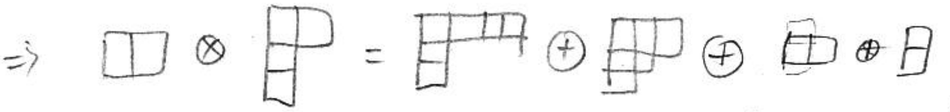
SU(n)
дим.
проп
SU(3) 8·8

$\dim\left(\begin{array}{llll} 4&5&6&7\\ 3&4&& \\ 2&3&& \end{array}\right) = \frac{4\cdot\cancel{5}\cdot\cancel{6}\cdot 7\cdot 3\cdot\cancel{4}\cdot\cancel{2}\cdot\cancel{3}}{\cancel{6}\cdot\cancel{5}\cdot\cancel{2}\cdot 1\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{2}\cdot 1} = 12\cdot 7 = 84$

$\dim\left(\begin{array}{ll} 4&5\\ 3& \\ 2& \end{array}\right) = \frac{\cancel{4}\cdot 5\cdot 3\cdot\cancel{2}}{\cancel{4}\cdot 1\cdot\cancel{2}\cdot 1} = 15$

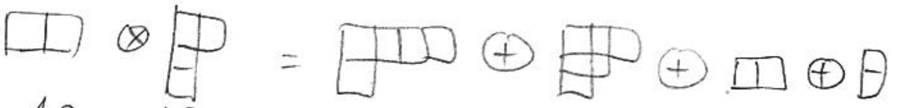
c) SU(4)





$\dim\left(\begin{array}{llll} 4&5&6&7\\ 3&&& \\ 2&&& \end{array}\right) = \frac{4\cdot 5\cdot\cancel{6}\cdot 7\cdot\cancel{3}\cdot\cancel{2}}{\cancel{6}\cdot\cancel{3}\cdot\cancel{2}\cdot 1\cdot 2\cdot 1} = 70$; $\dim\left(\begin{array}{lll} 4&5&6\\ 3&4& \\ 2&& \end{array}\right) = \frac{4\cdot\cancel{5}\cdot\overset{2}{\cancel{6}}\cdot\cancel{3}\cdot 4\cdot 2}{\cancel{5}\cdot\cancel{3}\cdot 1\cdot\cancel{3}\cdot 1\cdot 1} = 16\cdot 4 = 64$

$\dim\left(\begin{array}{l} 4\\ 3 \end{array}\right) = \frac{4\cdot 3}{2\cdot 1} = 6$; $\dim\left(\begin{array}{ll} 4&3 \end{array}\right) = \frac{4\cdot 5}{2\cdot 1} = 10$; $\dim\left(\begin{array}{ll} 4&5\\ 3& \\ 2& \end{array}\right) = \frac{\cancel{4}\cdot 5\cdot 3\cdot\cancel{2}}{\cancel{4}\cdot 1\cdot\cancel{2}\cdot 1} = 15$



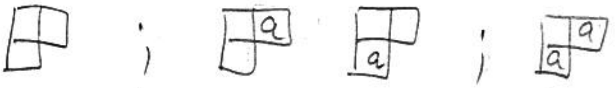
dim: $10 \cdot 15 = 70 + 64 + 10 + 6 = 150$ ✓

а) Разложити SU(3) октет ~~на~~ дендитет на изоспинске подмултиплете (мултиплете SU(2) групе)

SU(3) $\begin{array}{ll} 3&4\\ 2& \end{array}$ $\frac{\cancel{3}\cdot 4\cdot 2}{\cancel{3}\cdot 1\cdot 1} = 8$

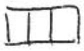
прочитамо као SU(2) шему, па убацујемо а на све могуће начине да добијемо опет шему, затим за ...

SU(2) :



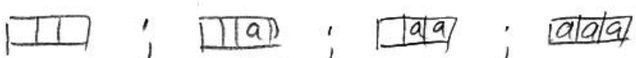
$\dim(\bullet) = 1$; $\dim(\square) = 2$; $\dim\left(\begin{array}{ll} 2&3 \end{array}\right) = \frac{2\cdot 3}{2} = 3$ $\qquad 1 + 2\cdot 2 + 3 = 8$ ✓

SU(3)



$\dim\left(\begin{array}{lll} 3&4&5 \end{array}\right) = \frac{\cancel{3}\cdot\overset{2}{\cancel{4}}\cdot 5}{\cancel{3}\cdot\cancel{2}} = 10$

SU(2) :



$\dim\left(\begin{array}{lll} 2&3&4 \end{array}\right) = \frac{\cancel{2}\cdot\cancel{3}\cdot 4}{\cancel{3}\cdot\cancel{2}} = 4$ $\qquad 4 + 3 + 2 + 1 = 10$ ✓

б) Разложити следеће репрезентације SU(4) групе на мултиплете SU(3) групе:

$\ydiagram{2,2}$ и $\ydiagram{2,1}$

SU(4) : $\dim \begin{ytableau} 4 & 5 \\ 3 & 4 \end{ytableau} = \frac{4\cdot 5\cdot \cancel{3}\cdot \cancel{4}}{\cancel{3}\cdot \cancel{2}\cdot \cancel{2}\cdot 1} = 20$

SU(3) : $\ydiagram{2,2}$; $\ytableaushort{{}{},{}a}$; $\ytableaushort{{}{},aa}$; Не може више од 2 a због антисим.

$\dim\left(\begin{ytableau} 3 & 4 \\ 2 & 3 \end{ytableau}\right) = \frac{3\cdot 4\cdot \cancel{2}\cdot \cancel{3}}{\cancel{3}\cdot \cancel{2}\cdot 2\cdot 1} = 6$; $\dim\left(\begin{ytableau} 3 & 4 \\ 2 \end{ytableau}\right) = \frac{\cancel{3}\cdot 4\cdot 2}{\cancel{3}\cdot 1\cdot 1} = 8$; $\dim\left(\begin{ytableau} 3 & 4 \end{ytableau}\right) = \frac{3\cdot \overset{2}{\cancel{4}}}{\cancel{2}} = 6$

$$20 = 6 + 8 + 6 \qquad \underbrace{\ydiagram{2,2} \oplus \ydiagram{2,1} \oplus \ydiagram{2}}_{\text{SU(3) подмултиплети од SU(4) } \ydiagram{2,2}}$$

SU(4) : $\dim \begin{ytableau} 4 & 5 \\ 3 \end{ytableau} = \frac{4\cdot 5\cdot 3}{\cancel{3}\cdot 1\cdot 1} = 20$

SU(3) $\ydiagram{2,1}$; $\ytableaushort{{}a,{}}$ $\ytableaushort{{}{},a}$; $\ytableaushort{{}a,a}$

$\dim\left(\begin{ytableau} 3 & 4 \\ 2 \end{ytableau}\right) = \frac{\cancel{3}\cdot 4\cdot 2}{\cancel{3}\cdot 1\cdot 1} = 8$; $\dim\left(\begin{ytableau} 3 \\ 2 \end{ytableau}\right) = \frac{3\cdot 2}{2} = 3$; $\dim\left(\begin{ytableau} 3 & 4 \end{ytableau}\right) = \frac{3\cdot 4}{2} = 6$; $\dim \ydiagram{1} = 3$

$$20 = 8+3+6+3 \qquad \underbrace{\ydiagram{2,1} \oplus \ydiagram{1,1} \oplus \ydiagram{2} \oplus \ydiagram{1}}_{\text{SU(3) подмултиплети од SU4) } \ydiagram{2,1}}$$

а) Користећи Јунгове шеме у оквиру SU(3) „flavor" симетрије, нацртати тежинске дијаграме за 3, 3*, 8, 10-димензионалну репрезентацију.

б) Користећи се овим резултатом одредити SU(3) таласне ф-је октета псеудоскаларних мезона и барионског декуплета.

⊛ У којим репрезентацијама се налазе бариони и мезони у оквиру SU(5) кварк модела?

бариони $qqq \Rightarrow (\ydiagram{1} \otimes \ydiagram{1}) \otimes \ydiagram{1} = (\ydiagram{2} \oplus \ydiagram{1,1}) \otimes \ydiagram{1} = \ydiagram{3} \oplus \ydiagram{2,1} + \ydiagram{2,1} \oplus \ydiagram{1,1,1}$

$= \ydiagram{3} \oplus 2\,\ydiagram{2,1} \oplus \ydiagram{1,1,1}$

$\dim\left(\ytableaushort{567}\right) = \frac{5\cdot\cancel{6}\cdot 7}{\cancel{3\cdot 2}\cdot 1} = 35$ $\qquad 5\cdot 5\cdot 5 = 125 = 35 + 2\cdot 40 + 10$ ✓

$\dim\left(\ytableaushort{56,4}\right) = \frac{5\cdot 6\cdot 4}{3\cdot 1\cdot 1} = 40$

$\dim\left(\ytableaushort{5,4,3}\right) = \frac{5\cdot 4\cdot \cancel{3}}{\cancel{3}\cdot 2\cdot 1} = 10$

Бариони се налазе у 35-dim, 2 40-дим. и 10-дим. репрезентацији

мезони $q\bar{q} \Rightarrow \ydiagram{1} \otimes \ydiagram{1,1,1,1} = \ydiagram{2,1,1,1} \oplus \bullet$

$\dim\left(\ytableaushort{56,4,3,2}\right) = \frac{\cancel{5}\cdot 6\cdot 4\cdot \cancel{3}\cdot\cancel{2}}{\cancel{5}\cdot\cancel{3}\cdot\cancel{2}} = 24$ $\qquad 5\cdot 5 = 25 = 24+1$

Мезони се налазе у 24-дим. и синглетној репрезентацији.

⊛ Одредити у којим репрезентацијама се налазе бариони и мезони у оквиру SU(4) кварк модела. Користећи добијени резултат проверити да ли се у судару бариона из 20-дим. репрезентације и мезона из 15-дим. репрезентације, могу добити барион из 20* димензионе или мезон из 15-дим.

бариони: $qqq \quad \ydiagram{1} \otimes \ydiagram{1} \otimes \ydiagram{1} = (\ydiagram{2} \oplus \ydiagram{1,1}) \otimes \ydiagram{1} = \ydiagram{3} \oplus 2\,\ydiagram{2,1} \oplus \ydiagram{1,1,1}$

$\dim \ytableaushort{456} = \frac{4\cdot 5\cdot 6}{\cancel{3\cdot 2}} = 20$; $\dim \ytableaushort{45,3} = \frac{4\cdot 5\cdot 3}{\cancel{3}\cdot 1\cdot 1} = 20$; $\dim \ydiagram{1,1,1} = 4$

$4\cdot 4\cdot 4 = 64 = 20 + 20\cdot 2 + 4$

налазе се у 20 дим, 20* дим и 4*-дим.

мезони: $q\bar{q} \quad \ydiagram{1} \otimes \ydiagram{1,1,1} = \ydiagram{2,1,1} \oplus \bullet$

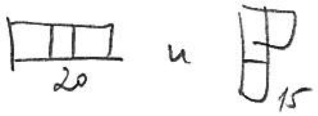
$\dim \ytableaushort{45,3,2} = \frac{\cancel{4}\cdot 5\cdot 3\cdot\cancel{2}}{\cancel{4\cdot 2}} = 15$

$4\cdot 4 = 15+1$

налазе се у 15 дим. и синглетној репрезентацији.

$B_{20} + M_{15} \longrightarrow B_{20^*} + M_{15}$

Множимо репрез.

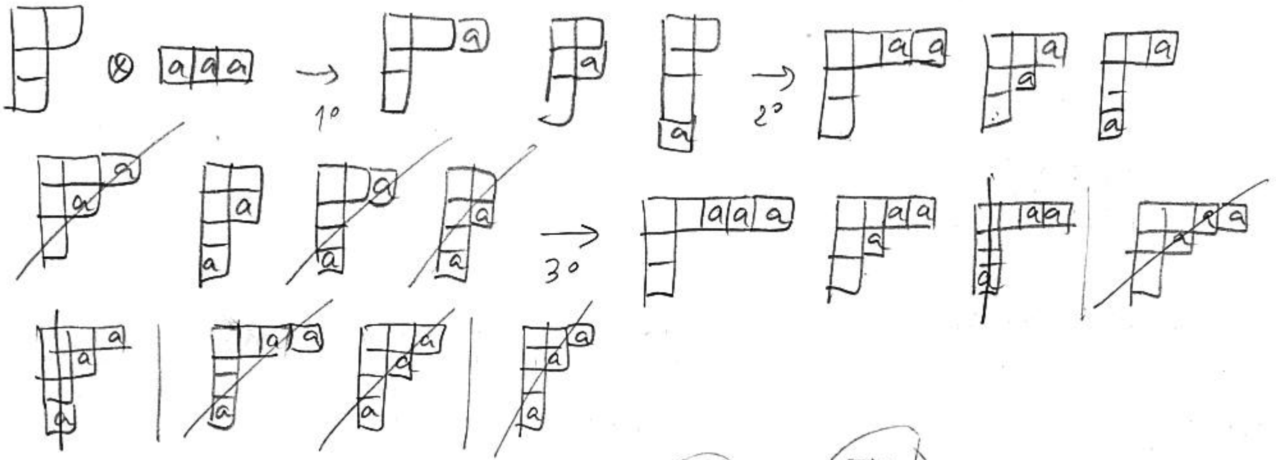


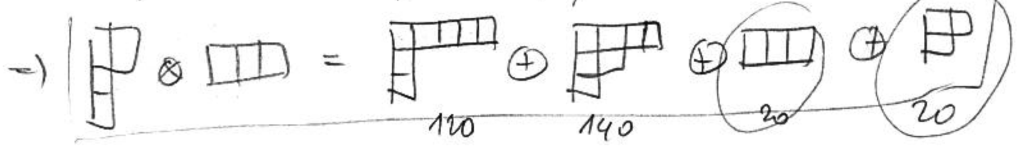
→ разложимо на ире

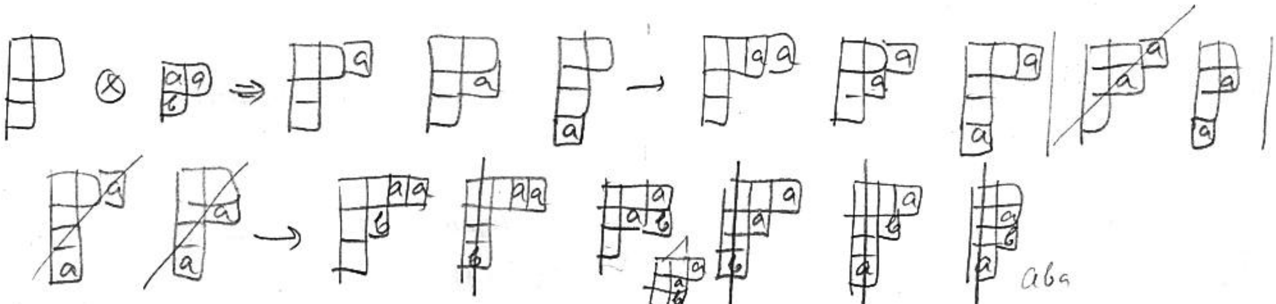
— || —

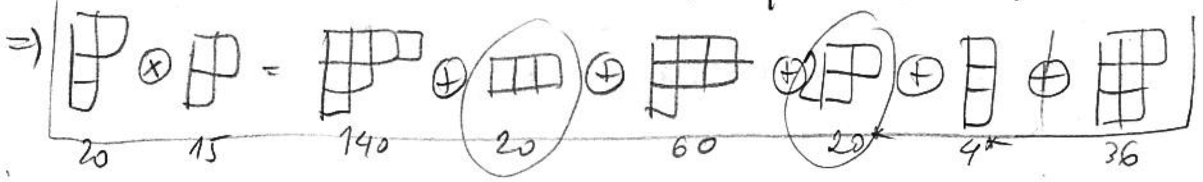
→ разложимо на ире

Ако има заједничких репрезентација у разлагању одговор је потврдан.









могуће је!

Користећи Јангове шеме у оквиру SU(3) „flavour" групе симетрије, нацртати тежинске дијаграме за 3, 3*, 8, и 10-дим. репрезентације.

<u>SU(3) група</u> $\square$ - фунд. репрезент. - 3 дим.

su(3) алгебра Гелманове матрице $\left[\frac{\lambda_a}{2}, \frac{\lambda_b}{2}\right] = i f_{abc} \frac{\lambda_c}{2}$

Картанова подалгебра: $H_1 = \frac{1}{\sqrt{6}} \lambda_3$; $H_2 = \frac{1}{\sqrt{6}} \lambda_8$

$$H_1 = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix} \qquad H_2 = \frac{1}{3\sqrt{2}} \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix}$$

фунд. репр. $u_1 = \boxed{1}$; $u_2 = \boxed{2}$; $u_3 = \boxed{3}$ базисни вектори

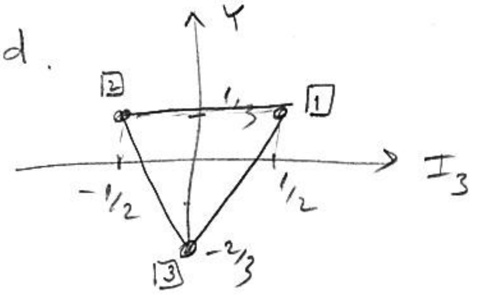
тежине $\vec{H} u_1 = \underbrace{\begin{pmatrix} \frac{1}{\sqrt{6}} \\ \frac{1}{3\sqrt{2}} \end{pmatrix}}_{\vec{m}_1} u_1$; $\vec{H} u_2 = \underbrace{\begin{pmatrix} -\frac{1}{\sqrt{6}} \\ \frac{1}{3\sqrt{2}} \end{pmatrix}}_{\vec{m}_2} u_2$; $\vec{H} u_3 = \underbrace{\begin{pmatrix} 0 \\ -\frac{\sqrt{2}}{3} \end{pmatrix}}_{\vec{m}_3} u_3$

тежине

Користи се други базис: $I_3 = \frac{\sqrt{6}}{2} m_1$; $Y = \sqrt{2} m_2$

$\underset{\boxed{1}}{\vec{m}_1} = \begin{pmatrix} 1/2 \\ 1/3 \end{pmatrix}$; $\underset{\boxed{2}}{\vec{m}_2} = \begin{pmatrix} -1/2 \\ 1/3 \end{pmatrix}$ $\underset{\boxed{3}}{\vec{m}_3} = \begin{pmatrix} 0 \\ -2/3 \end{pmatrix}$ $\begin{pmatrix} I_3 \\ Y \end{pmatrix}$ ← 3. комп. изоспина ← хипернабој

- $\square$ - 3d.



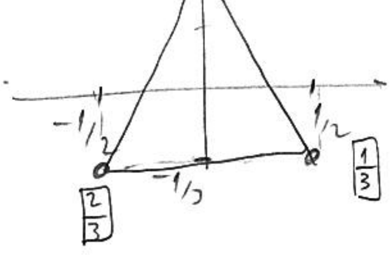
станд. уређење → не опада
↓ расте

- $\begin{array}{|c|}\hline \\ \hline \\ \hline \end{array} = 3^*$ $\begin{array}{|c|}\hline 1 \\ \hline 2 \\ \hline \end{array}$, $\begin{array}{|c|}\hline 1 \\ \hline 3 \\ \hline \end{array}$, $\begin{array}{|c|}\hline 2 \\ \hline 3 \\ \hline \end{array}$

тежине су адитивне!

$\vec{m}\left(\begin{array}{|c|}\hline 1 \\ \hline 2 \\ \hline \end{array}\right) = \vec{m}(\boxed{1}) + \vec{m}(\boxed{2}) = \begin{pmatrix} 0 \\ 2/3 \end{pmatrix}$ $\vec{m}\left(\begin{array}{|c|}\hline 1 \\ \hline 3 \\ \hline \end{array}\right) = \vec{m}(\boxed{1}) + \vec{m}(\boxed{3}) = \begin{pmatrix} 1/2 \\ -1/3 \end{pmatrix}$

$\vec{m}\left(\begin{array}{|c|}\hline 2 \\ \hline 3 \\ \hline \end{array}\right) = \vec{m}(\boxed{2}) + \vec{m}(\boxed{3}) = \begin{pmatrix} -1/2 \\ -1/3 \end{pmatrix}$

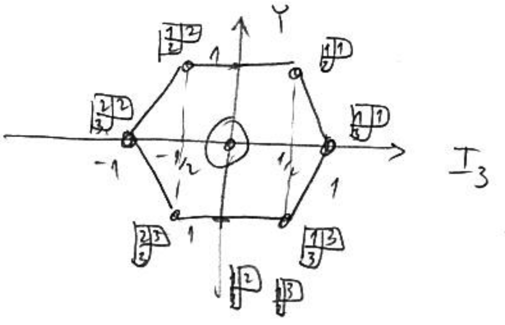


8 d репрезентација $\ydiagram{2,1}$ $\dim\left(\begin{ytableau}3&4\\2\end{ytableau}\right) = \frac{3\cdot4\cdot2}{3\cdot1\cdot1} = 8$

$\longrightarrow$ неопада.

$\begin{ytableau}1&1\\2\end{ytableau}$ $\begin{ytableau}1&1\\3\end{ytableau}$ $\begin{ytableau}1&2\\2\end{ytableau}$ $\begin{ytableau}1&2\\3\end{ytableau}$ $\begin{ytableau}1&3\\2\end{ytableau}$ $\begin{ytableau}1&3\\3\end{ytableau}$ $\begin{ytableau}2&2\\3\end{ytableau}$ $\begin{ytableau}2&3\\3\end{ytableau}$ $\downarrow$ расте

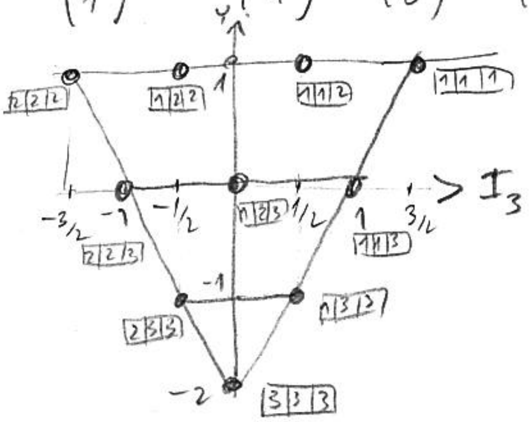
$\begin{pmatrix}1/2\\1\end{pmatrix}$ $\begin{pmatrix}1\\0\end{pmatrix}$ $\begin{pmatrix}-1/2\\1\end{pmatrix}$ $\begin{pmatrix}0\\0\end{pmatrix}$ $\begin{pmatrix}0\\0\end{pmatrix}$ $\begin{pmatrix}1/2\\-1\end{pmatrix}$ $\begin{pmatrix}-1\\0\end{pmatrix}$ $\begin{pmatrix}-1/2\\-1\end{pmatrix}$ $\longrightarrow$ $\begin{pmatrix}I_3\\Y\end{pmatrix}$



10 d репрезентација $\ydiagram{3}$ $\dim\left(\begin{ytableau}3&4&5\end{ytableau}\right) = \frac{\cancel{3}\cdot4\cdot5}{\cancel{3}\cdot2} = 10$

$\begin{ytableau}1&1&1\end{ytableau}$ $\begin{ytableau}1&1&2\end{ytableau}$ $\begin{ytableau}1&1&3\end{ytableau}$ $\begin{ytableau}1&2&2\end{ytableau}$ $\begin{ytableau}1&2&3\end{ytableau}$ $\begin{ytableau}1&3&3\end{ytableau}$ $\begin{ytableau}2&2&2\end{ytableau}$ $\begin{ytableau}2&2&3\end{ytableau}$ $\begin{ytableau}2&3&3\end{ytableau}$ $\begin{ytableau}3&3&3\end{ytableau}$

$\begin{pmatrix}\frac{3}{2}\\1\end{pmatrix}$ $\begin{pmatrix}1/2\\1\end{pmatrix}$ $\begin{pmatrix}1\\0\end{pmatrix}$ $\begin{pmatrix}-1/2\\1\end{pmatrix}$ $\begin{pmatrix}0\\0\end{pmatrix}$ $\begin{pmatrix}\frac{1}{2}\\-1\end{pmatrix}$ $\begin{pmatrix}-3/2\\1\end{pmatrix}$ $\begin{pmatrix}-1\\0\end{pmatrix}$ $\begin{pmatrix}-1/2\\-1\end{pmatrix}$ $\begin{pmatrix}0\\-2\end{pmatrix}$



У оквиру SU(3) кварк модела:

бариони $\underset{q}{\square} \otimes \underset{q}{\square} \otimes \underset{q}{\square} = \left(\ydiagram{2} \oplus \ydiagram{1,1}\right) \otimes \square = \ydiagram{2,1} \oplus \ydiagram{3} \oplus \ydiagram{2,1} \oplus \ydiagram{1,1,1}$

$= \ydiagram{3} \oplus 2\,\ydiagram{2,1} \oplus \bullet$

мезони $\underset{q}{\square} \otimes \underset{\bar{q}}{\ydiagram{1,1}} = \ydiagram{2,1} \oplus \ydiagram{1,1,1} = \ydiagram{2,1} \oplus \bullet$

Мултиплет – исти спин и парност, различито наелектрисање

$Q = I_3 + \frac{Y}{2}$ Гел Ман, Нишиџима

$S = Y - B$ странност

У SU(3) кварк моделу базисни вектори су кваркови $\boxed{1} = u$; $\boxed{2} = d$; $\boxed{3} = s$

	Q	I	I_3	Y	B	S
u	$2/3$	$1/2$	$1/2$	$1/3$	$1/3$	0
d	$-1/3$	$1/2$	$-1/2$	$1/3$	$1/3$	0
s	$-1/3$	0	0	$-2/3$	$1/3$	-1

(постоје и t, b, c кваркови SU(6)
кварк модел)

б) октет SU(3) $\ydiagram{2,1}$
барионски

$$\underbrace{\psi^{\{j[i\}k]}}_{\text{ово је 8d реп.}} = A_{13} S_{12} \psi^{ijk} \leftarrow \ydiagram{1,1} \otimes \ydiagram{1} \quad AS$$

$\boxed{I}$ ⊗⊗ SA

$$\ytableaushort{11,2} \sim A_{13} S_{12}\, uud$$

$$= (1-P_{13})(1+P_{12})\, uud = (1-P_{13}) \cdot 2uud = 2(uud - duu) \Rightarrow \boxed{\ytableaushort{11,2} = \frac{1}{\sqrt{2}}(uud - duu)}$$

Σu

$$\ytableaushort{11,3} \sim A_{13} S_{12}\, uus = 2(uus - suu) \Rightarrow \boxed{\ytableaushort{11,3} = \frac{1}{\sqrt{2}}(uus - suu)}$$

$$\ytableaushort{12,2} \sim A_{13} S_{12}\, udd = A_{13}(udd + dud) = udd - ddu + \cancel{dud} - \cancel{dud} ; \boxed{\ytableaushort{12,2} \sim \frac{1}{\sqrt{2}}(udd - ddu)}$$

$$\ytableaushort{12,3} \sim A_{13} S_{12}\, uds = A_{13}(uds + dus) = uds - sdu + dus - sud$$

$$\boxed{\ytableaushort{12,3} = \frac{1}{2}(uds - sdu + dus - sud)} \checkmark \quad \Sigma^0$$

$$\ytableaushort{13,2} \sim A_{13} S_{12}\, usd = A_{13}(usd + sud) = usd - dsu + sud - dus$$

$$? \quad \boxed{\ytableaushort{13,2} = \frac{1}{2}(usd - dsu + sud - dus)} \qquad \Lambda$$

$$\ytableaushort{13,3} \sim A_{13} S_{12}\, uss = A_{13}(uss + sus) = uss - ssu + \cancel{sus} - \cancel{sus} \Rightarrow \boxed{\ytableaushort{13,2} \simeq \frac{1}{\sqrt{2}}(uss - ssu)}$$

$$\ytableaushort{22,3} \sim A_{13} S_{12}\, dds = A_{13}\, 2dds = 2(dds - sdd) \rightarrow \boxed{\ytableaushort{22,3} = \frac{1}{\sqrt{2}}(dds - sdd)}$$

$$\ytableaushort{23,3} \sim A_{13} S_{12}\, dss = A_{13}(dss + sds) = dss - ssd + \cancel{sds} - \cancel{sds} \Rightarrow \boxed{\ytableaushort{23,3} = \frac{1}{\sqrt{2}}(dss - ssd)}$$

декуплет SU(3) $\ydiagram{3}$ $\quad \psi^{\{ijk\}} = (1 + P_{12} + P_{13} + P_{23} + P_{13}P_{12} + P_{12}P_{13})\,\psi^{ijk}$

$\ytableaushort{111} = uuu$

$\ytableaushort{112} = \frac{1}{\sqrt{3}}(uud + udu + duu)$

$\ytableaushort{113} = \frac{1}{\sqrt{3}}(uus + usu + suu)$

$\ytableaushort{122} = \frac{1}{\sqrt{3}}(udd + dud + ddu)$

$\ytableaushort{123} = \frac{1}{\sqrt{6}}(uds + dus + sdu + usd + sud + dsu)$

$\ytableaushort{133} = \frac{1}{\sqrt{3}}(uss + sus + ssu)$

$\ytableaushort{222} = ddd$

$\ytableaushort{223} = \frac{1}{\sqrt{3}}(dds + dsd + sdd)$

$\ytableaushort{233} = \frac{1}{\sqrt{3}}(dss + sds + ssd)$

$\ytableaushort{333} = sss$

На основу квантних бројева, одредити кварк структуру следећих честица :

$Q = I_3 + \frac{Y}{2}$; $S = Y - B$

$\begin{pmatrix} I_3 \\ Y \end{pmatrix}$ Δ^{++} : $\begin{pmatrix} 3/2 \\ 1 \end{pmatrix}$ $\Rightarrow$ $Q = 2$; $B(\Delta^{++}) = 1$ $S = 1 - 1 = 0$ → нема s кварка!

барион → qqq $Q(u) = \frac{2}{3}$; $Q(d) = Q(s) = \frac{-1}{3}$ $\Rightarrow$ $\Delta^{++} = uuu$

π^0 : $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\Rightarrow$ $Q = 0$ $B = 0$; $S = 0$

мезон → $q\bar{q}$ $\pi^0 \sim u\bar{u}, d\bar{d}, \cancel{s\bar{s}}$ енитерм.

$\bar{K}^0$: $\begin{pmatrix} 1/2 \\ -1 \end{pmatrix}$ $\Rightarrow$ $Q = 0$; $B = 0$; $S = -1 - 0 = -1$ → има s кварк!

мезон $\bar{K}^0 = \bar{u}s$ $Q(\bar{u}) = -\frac{2}{3})$ $Q(s) = -\frac{1}{3}$ $S(s) = -1$!

Σ^{*+} : $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\Rightarrow$ $Q = 1$; $B = 1$; $S = -1$ има s кварк

барион qqq $\Sigma^{*+} = uus$ $Q(u) = \frac{2}{3}$ $Q(s) = -\frac{1}{3}$

Ω^- : $\begin{pmatrix} -0 \\ -2 \end{pmatrix}$ $\Rightarrow$ $Q = -2$; $B = 1$; $S = -3$ има 3 s кварка

барион qqq $\Rightarrow$ $\Omega^- = sss$

Λ^0 : $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\Rightarrow$ $Q = 0$; $B = 1$, $S = 0 - 1 = -1$ има s кварк

барион qqq $\Lambda^0 = uds$

Ξ^- : $\begin{pmatrix} -1/2 \\ -1 \end{pmatrix}$ $\Rightarrow$ $Q = -1$; $B = 1$; $S = -1 - 1 = -2$ – два s кварка

барион qqq: $\Xi^- = dss$ $Q(d) = Q(s) = -\frac{1}{3})$

Σ^0 : $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\Rightarrow$ $Q = 0$; $B = 1$; $S = -0 - 1$ има 1 s кварк

барион : $\Sigma^0 = uds$

K^- : $\begin{pmatrix} -1/2 \\ -1 \end{pmatrix}$ $\Rightarrow$ $Q = -1$; $B = 0$ $S = -1 - 0 = -1$ има s кварк

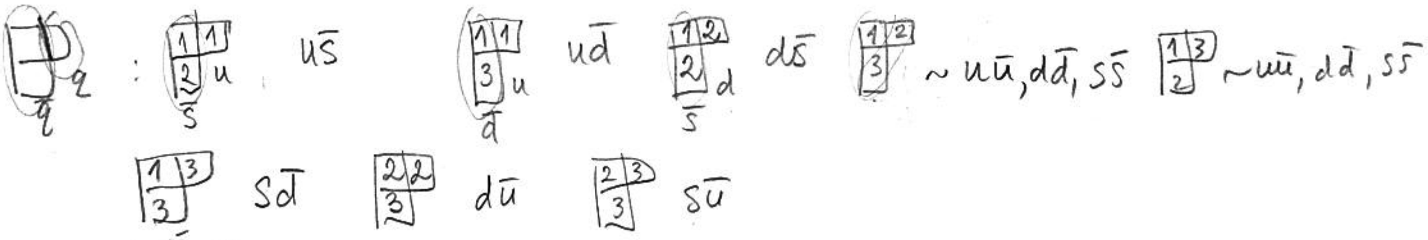
мезон $q\bar{q}$ $K^- = \bar{u}s$ $Q(\bar{u}) = -\frac{2}{3}$; $Q(s) = -\frac{1}{3}$

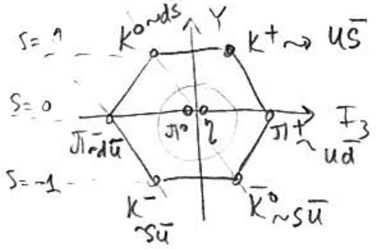
p : $\begin{pmatrix} 1/2 \\ 1 \end{pmatrix}$ $\Rightarrow$ $Q = 1$; $B = 1$; $S = 1 - 1 = 0$

барион qqq $p = uud$ $Q(u) = \frac{2}{3}$ $Q(d) = \frac{-1}{3}$

2.6. б) октет псеудоскаларних мезона

састоји се из q и $\bar q$ репрез. $\square$ и $\bar\square = \boxminus$ две фунд. репрезент. SU(3) групе





η' - синглет $\begin{array}{|c|}\hline 1\\\hline 2\\\hline 3\\\hline\end{array} = \frac{1}{\sqrt3}(u\bar u + d\bar d + s\bar s)$

$\langle \eta|\eta'\rangle = \langle \pi^0|\eta'\rangle = \langle \eta|\pi^0\rangle = 0$

$\eta = \alpha u\bar u + \beta d\bar d + \gamma s\bar s$; $\pi^0 = \alpha' u\bar u + \beta' d\bar d + \cancelto{0}{\gamma' s\bar s}$ | експер. пиони су SU(2) триплет

$\langle \eta|\eta'\rangle = (\alpha+\beta+\gamma)\frac{1}{\sqrt3} = 0$ $\quad \langle \pi^0|\eta'\rangle = \frac{1}{\sqrt3}(\alpha'+\beta') = 0$; $\langle \eta|\pi^0\rangle = \alpha\alpha' + \beta\beta' = 0$

$\alpha' = -\beta' \Rightarrow \boxed{\pi^0 = \frac{1}{\sqrt2}(u\bar u - d\bar d)}$ $\quad \alpha = \beta$; $\gamma = -2\alpha$; $\boxed{\eta = \frac{1}{\sqrt6}(u\bar u + d\bar d - 2s\bar s)}$

\+ Да ли су могући следећи процеси? Преко којих интеракција се одвијају?

Слаба интеракција: лептони и бариони; очувано Q, B, L_e, L_μ, L_τ

јака интеракција: бариони; очувано Q, B, I_3, S.

$Q = I_3 + \frac{Y}{2}$; $S = Y - B$

$\pi^- + p \longrightarrow \Delta^0$

$Q: -1 + 1 = 0$ ✓

$B: 0 + 1 = 1$ ✓

$I_3: -1 + \frac12 = -\frac12$ ✓

$S: 0 + 0 = 0$ ✓

могућ процес, јака интеракција

$\mu^- \rightarrow e^+ + \nu_e + \nu_\tau$ (слаба интеракција)

$Q: -1 \neq 1 + 0 + 0$ ×

$L_e \;\; 0 = -1 + 1 + 0$ ✓

$L_\mu \;\; 1 \neq 0 + 0 + 0$ ×

$L_\tau \;\; 0 \neq 0 + 0 + 1$ ×

немогућ процес

$\mu^- \rightarrow \nu_\mu + \tau^- + \bar\nu_\tau$ (слаба интеракција)

$Q: -1 = 0 - 1 + 0$ ✓

$L_\mu \;\; 1 = 1 + 0 + 0$ ✓

$L_\tau \;\; 0 = 0 + 1 - 1$ ✓

кв. бројеви у реду али кинематички забрањен

$m(\mu^-) < m(\tau^-) + m(\nu_\mu) + m(\bar\nu_\tau)$

немогућ процес

$\Delta^+ \longrightarrow n + p$

$Q: 1 = 0 + 1$ ✓

$B: 1 \neq 1 + 1$ немогућ процес

$\Omega^- \longrightarrow \Xi^0 + \pi^-$ (слаба интеракција)

$Q: -1 = 0 - 1$ може по слабој!

$B: 1 = 1 + 0$

$S: -3 \neq -2$

$\pi^0 + p \rightarrow K^+ + K^0 + n$ (слаба)

$Q: 0 + 1 = 1 + 0 + 0$ ✓

$B: 0 + 1 = 0 + 0 + 1$ ✓ Може по слабој

$S: 0 + 0 \neq 1 + 1 + 0$ не може јака

$\pi^0 + n \rightarrow K^+ + K^-$

$Q: 0+0 = 0+0$ ✓

$B: 0+1 = 0+0$ X

. немогућ процес

$K^- + p \rightarrow \pi^0 + \Lambda^0$

$Q: -1 + 1 = 0+0$

$B: 0+1 = 0+1$

$S: -1+0 = 0-1$

$I_3: -\frac{1}{2} + \frac{1}{2} = 0+0$ може, јака интеракција

$\mu^+ \rightarrow e^+ + \gamma$

$L_\mu: 1 \neq 0+0$ X немогућ процес

$\Sigma^+ + p \rightarrow K^+ + p$

$Q: 1+1 = 1+1$ ✓

$B: 1+1 \neq 0+1$ X немогућ процес

(*) На основу закона одржања допунити следеће процесе:

$\pi^- \rightarrow ? + \bar{\nu}_\mu$ (слаба интеракција – лептона)

$Q: -1 = \boxed{-1} + 0$

$B: 0 = \boxed{0} + 0$

$L_\mu: 0 = \boxed{1} - 1$

$? = \mu^-$

$\boxed{\pi^- \rightarrow \mu^- + \bar{\nu}_\mu}$

$\tau^- \rightarrow e^- + \boxed{?}_1 + \boxed{?}_2$

$Q: -1 = -1 + a - a$

$B: 0 = 0 + b - b$

$L_\tau: 1 = 0 + 1 + 0$

$L_e: 0 = 1 + 0 \boxed{-1}$

$?_1 = \tau^-$ или ν_τ

$?_2 = e^+$ или $\bar{\nu}_e$

$\tau^- \rightarrow e^- + \nu_\tau + \bar{\nu}_e$

$\tau^- \rightarrow e^- + e^+ + \tau^-$ X немогуће због маса.

$e^- + p^+ \rightarrow \nu_e + ?$

$Q: -1 + 1 = 0 + \boxed{0}$

$B: 0 + 1 = 0 + \boxed{1}$

$L_e: 1 + 0 = 1 + \boxed{0}$

барион неутралан. n

$n \rightarrow p^+ + e^- + \bar{\nu}_e$ кросинг симетрија $\Rightarrow n + \nu_e$ · $p^+ + e^-$

$\Rightarrow \boxed{e^- + p^+ \rightarrow \nu_e + n}$

Не може нешто друго због маса.

$\overset{ddd}{\Delta^-} \rightarrow \overset{udd}{n} + ?$

$Q: -1 = 0 + \boxed{-1}$

$B: 1 = 1 + \boxed{0}$

$S: 0 = 0 + \boxed{0}$

$I_3: -\frac{3}{2} = -\frac{1}{2} \boxed{-1}$

Ако је јака интеракција π^-, K^- не може због маса

$\boxed{\Delta^- \rightarrow n + \pi^-}$ јака

$\Sigma^- + ? \rightarrow \Lambda^0 + n$

$Q: -1 + \boxed{1} = 0 + 0$

$S: -1 + 0 = -1 + 0$

$? = p$

јака интер.

$\boxed{\Sigma^- + p \rightarrow \Lambda^0 + n}$