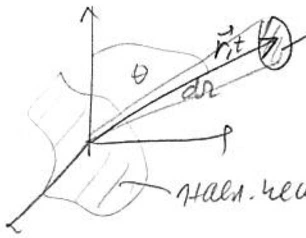
$$\boxed{\tau = \frac{i\omega\varepsilon\mu_0}{k^2-\omega^2/c^2} E_{0,pq}\left(\frac{p\pi}{a}\right)(-1)^p \sin\left(\frac{q\pi}{b}y\right) e^{ikz-i\omega t}}$$

Зрачење наелектрисаних честица

$$I = \frac{1}{6\pi\varepsilon_0 c^3}\left|\ddot{\vec p}(\tau)\right|^2 \qquad \tau = t-\frac{r}{c}$$ ретардовано време ; Ларморова ф-ла



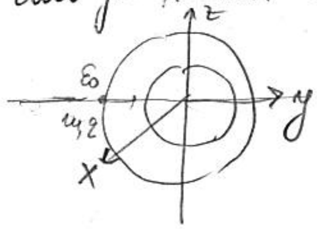
$d\vec S = \vec n\, dS = \vec n\, r^2 d\Omega$

I – израчена снага у свим правцима

$$dI = \frac{1}{4\pi\varepsilon_0 c^3}\frac{\left(\ddot{\vec p}(\tau)\times\vec n\right)^2}{4\pi}\,d\Omega$$

израчено у угао $d\Omega$

Честица масе m и наелектрисања q пролеће по пречнику кроз куглу полупречника R, која је равномерно запремински наелектрисана супротним наелектрисањем Q. Одреди енергију коју честица израчи за време проласка кроз куглу у диполној апроксимацији ако је почетна кинетичка енергија честице била $\tilde{E}_0$.



$$\frac{d\mathcal{E}}{dt} = \frac{1}{6\pi\varepsilon_0 c^3}\left|\ddot{\vec p}(\tau)\right|^2 \qquad \tau = t-\frac{r}{c}\ ,\ \dot{}\ \text{– извод по } \tau!$$

$r \approx$ константно за диполну апроксим. па је извод по τ извод по t.

$\vec E(\vec r) = E(r)\vec e_r$ од кугле ; $E(r)\cdot 4r^2\pi = \frac{1}{\varepsilon_0}\frac{Q}{\frac{4}{3}R^3\pi}\frac{4}{3}r^3\pi \Rightarrow \boxed{\vec E(r) = \frac{Q\vec r}{4\pi\varepsilon_0 R^3}}$

II Њутнов закон $m\ddot{\vec r} = q\vec E = \frac{qQ\vec r}{4\pi\varepsilon_0 R^3}$ дуж пречника $\vec e_y$ $\boxed{\vec r = y\vec e_y}$

$m\ddot y - \frac{qQ}{4\pi\varepsilon_0 R^3}y = 0$; $\ddot y - \underbrace{\frac{qQ}{4\pi\varepsilon_0 R^3 m}}_{\omega^2} y = 0$; $\boxed{\omega^2 = -\frac{qQ}{4\pi\varepsilon_0 R^3 m}} > 0$

$y(t) = A\cos\omega t + B\sin\omega t$

$y(t=0) = -R = A$; $\dot y(t=0) = \sqrt{\frac{2\tilde E_0}{m}} \leftarrow \frac{mv^2}{2} = \tilde E_0$

$$A = R \qquad \dot{y}(t=0) = B\omega = \sqrt{\frac{2\tilde{\varepsilon}_0}{m}} \Rightarrow B = \frac{1}{\omega}\sqrt{\frac{2\tilde{\varepsilon}_0}{m}}$$

$$y(t) = -R\cos\omega t + \frac{1}{\omega}\sqrt{\frac{2\tilde{\varepsilon}_0}{m}}\sin\omega t$$

за $t=t_1$ $\quad y(t_1) = R = -R\cos\omega t_1 + \frac{1}{\omega}\sqrt{\frac{2\tilde{\varepsilon}_0}{m}}\sin\omega t_1$

$$R\underbrace{(1+\cos\omega t_1)}_{2\cos^2\frac{\omega t_1}{2}} = \frac{1}{\omega}\sqrt{\frac{2\tilde{\varepsilon}_0}{m}}\, 2\sin\frac{\omega t_1}{2}\cos\frac{\omega t_1}{2}$$

$$\cos\frac{\omega t_1}{2}\left(R\cos\frac{\omega t_1}{2} - \frac{1}{\omega}\sqrt{\frac{2\tilde{\varepsilon}_0}{m}}\sin\frac{\omega t_1}{2}\right) = 0 \Rightarrow \cos\frac{\omega t_1}{2} = 0 \text{ или } \boxed{\operatorname{tg}\frac{\omega t_1}{2} = R\omega\sqrt{\frac{m}{2\tilde{\varepsilon}_0}}}$$

$\omega t_1 = \pi$ или $\boxed{\omega t_1 = 2\operatorname{arctg}\left(R\omega\sqrt{\frac{m}{2\tilde{\varepsilon}_0}}\right)} < \pi$ ово решење нам треба

За то решење је брзина позитивна (од равнотежног положаја)

$$\dot{y}(t_1) = R\omega\sin\omega t_1 + \frac{1}{\cancel{\omega}}\sqrt{\frac{2\tilde{\varepsilon}_0}{m}}\cancel{\omega}\cos\omega t_1$$

$$= R\omega\,\frac{2R\omega\sqrt{\frac{m}{2\tilde{\varepsilon}_0}}}{1+R^2\omega^2\frac{m}{2\tilde{\varepsilon}_0}} + \sqrt{\frac{2\tilde{\varepsilon}_0}{m}}\,\frac{1-R^2\omega^2\frac{m}{2\tilde{\varepsilon}_0}}{1+R^2\omega^2\frac{m}{2\tilde{\varepsilon}_0}}$$

$$= \sqrt{\frac{2\tilde{\varepsilon}_0}{m}}\,\frac{1\cancel{-R^2\omega^2\frac{m}{2\tilde{\varepsilon}_0}}\cancel{+2\omega^2R^2\frac{m}{2\tilde{\varepsilon}_0}}}{1+R^2\omega^2\frac{m}{2\tilde{\varepsilon}_0}} = \sqrt{\frac{2\tilde{\varepsilon}_0}{m}} \quad W \text{ у реду је!}$$

$$\operatorname{tg}^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{1+\cos\alpha} \qquad (1+\cos\alpha)\operatorname{tg}^2\frac{\alpha}{2} = 1-\cos\alpha$$

$$\boxed{\cos\alpha = \frac{1-\operatorname{tg}^2\alpha/2}{1+\operatorname{tg}^2\alpha/2}}$$

$$\sin\alpha = \sqrt{1-\left(\frac{1-\operatorname{tg}^2\alpha/2}{1+\operatorname{tg}^2\alpha/2}\right)^2} = \sqrt{\frac{4\operatorname{tg}^2\alpha/2}{(1+\operatorname{tg}^2\alpha/2)^2}}$$

$$\sin\alpha = \frac{2\operatorname{tg}\alpha/2}{1+\operatorname{tg}^2\alpha/2}$$

$$\frac{d\mathcal{E}}{dt} = \frac{1}{6\pi\varepsilon_0 c^3}|\ddot{\vec{p}}(\tau)|^2 = \frac{1}{6\pi\varepsilon_0 c^3}|q\ddot{\vec{r}}|^2 = \frac{1}{6\pi\varepsilon_0 c^3}q^2\ddot{y}^2 = \frac{1}{6\pi\varepsilon_0 c^3}q^2\omega^4 y^2$$

$$= \frac{q^2\omega^4}{6\pi\varepsilon_0 c^3}\left(R^2\cos^2\omega t + \frac{1}{\omega^2}\frac{2\tilde{\varepsilon}_0}{m}\sin^2\omega t - \frac{2}{\omega}\sqrt{\frac{2\tilde{\varepsilon}_0}{m}}R\sin\omega t\cos\omega t\right)$$

$$\mathcal{E} = \frac{q^2\omega^4}{6\pi\varepsilon_0 c^3}\int_0^{t_1}\left(R^2\frac{1+\cos 2\omega t}{2} + \frac{\cancel{2}\tilde{\varepsilon}_0}{\omega^2 m}\frac{1-\cos 2\omega t}{\cancel{2}} - \frac{R}{\omega}\sqrt{\frac{2\tilde{\varepsilon}_0}{m}}\sin 2\omega t\right)dt$$

$$= \frac{q^2\omega^4}{6\pi\varepsilon_0 c^3}\left(\left(\frac{R^2}{2}+\frac{\tilde{\varepsilon}_0}{m\omega^2}\right)t_1 + \left(\frac{R^2}{2}-\frac{\tilde{\varepsilon}_0}{m\omega^2}\right)\frac{1}{2\omega}\cdot\sin 2\omega t_1 - \frac{R}{\omega}\sqrt{\frac{2\tilde{\varepsilon}_0}{m}}\frac{1}{2\omega}(1-\cos 2\omega t_1)\right)$$

$$t_1 = \frac{2}{\omega}\operatorname{arctg}\left(R\omega\sqrt{\frac{m}{2\tilde{\varepsilon}_0}}\right); \quad \sin 2\omega t_1 = 2\sin\omega t_1\cdot\cos\omega t_1 = \frac{4\operatorname{tg}\frac{\omega t_1}{2}\cdot\left(1-\operatorname{tg}^2\frac{\omega t_1}{2}\right)}{\left(1+\operatorname{tg}^2\frac{\omega t_1}{2}\right)^2} = \frac{4R\omega\sqrt{\frac{m}{2\tilde{\varepsilon}_0}}\left(1-R^2\omega^2\frac{m}{2\tilde{\varepsilon}_0}\right)}{\left(1+R^2\omega^2\frac{m}{2\tilde{\varepsilon}_0}\right)^2}$$

$$\cos 2\omega t_1 = 2\cos^2\omega t_1 - 1 = 2\left(\frac{1-\operatorname{tg}^2\frac{\omega t_1}{2}}{1+\operatorname{tg}^2\frac{\omega t_1}{2}}\right)^2 - 1 = 2\frac{\left(1-R^2\omega^2\frac{m}{2\tilde{\varepsilon}_0}\right)^2}{\left(1+R^2\omega^2\frac{m}{2\tilde{\varepsilon}_0}\right)^2} - 1$$

$$\mathcal{E} = \frac{q^2\omega^3R^2}{6\pi\varepsilon_0 c^3}\left(\left(1+\frac{2\tilde{\varepsilon}_0}{m\omega^2R^2}\right)\operatorname{arctg}R\omega\sqrt{\frac{m}{2\tilde{\varepsilon}_0}} + \left(1-\frac{2\tilde{\varepsilon}_0}{m\omega^2R^2}\right)\frac{R\omega\sqrt{\frac{m}{2\tilde{\varepsilon}_0}}\left(1-R^2\omega^2\frac{m}{2\tilde{\varepsilon}_0}\right)}{\left(1+R^2\omega^2\frac{m}{2\tilde{\varepsilon}_0}\right)^2} - \frac{1}{\cancel{2}\omega R}\sqrt{\frac{2\tilde{\varepsilon}_0}{m}}\cancel{2}\left(1-\frac{\left(1-R^2\omega^2\frac{m}{2\tilde{\varepsilon}_0}\right)^2}{\left(1+R^2\omega^2\frac{m}{2\tilde{\varepsilon}_0}\right)^2}\right)\right)$$

$$= \frac{q^2\omega^3R^2}{6\pi\varepsilon_0 c^3}\left[\left(1+\frac{2\tilde{\varepsilon}_0}{m\omega^2R^2}\right)\operatorname{arctg}R\omega\sqrt{\frac{m}{2\tilde{\varepsilon}_0}} + \frac{R\omega\sqrt{\frac{m}{2\tilde{\varepsilon}_0}}}{\left(1+R^2\omega^2\frac{m}{2\tilde{\varepsilon}_0}\right)^2}\underbrace{\left(1-\frac{2\tilde{\varepsilon}_0}{m\omega^2R^2}-R^2\omega^2\frac{m}{2\tilde{\varepsilon}_0}+1-4\right)}\right]$$

$$-\left(2+\frac{2\tilde{\varepsilon}_0}{m\omega^2R^2}+R^2\omega^2\frac{m}{2\tilde{\varepsilon}_0}\right) = -\frac{2\tilde{\varepsilon}_0}{m\omega^2R^2}\left(1+\left(\frac{R^2\omega^2m}{2\tilde{\varepsilon}_0}\right)^2+2\frac{R^2\omega^2m}{2\tilde{\varepsilon}_0}\right) = \frac{-2\tilde{\varepsilon}_0}{m\omega^2R^2}\left(1+\frac{R^2\omega^2m}{2\tilde{\varepsilon}_0}\right)^2$$

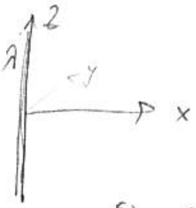
$$= \boxed{\frac{q^2\omega^3R^2}{6\pi\varepsilon_0 c^3}\left[\left(1+\frac{2\tilde{\varepsilon}_0}{m\omega^2R^2}\right)\operatorname{arctg}\left(R\omega\sqrt{\frac{m}{2\tilde{\varepsilon}_0}}\right) - \frac{1}{R\omega}\sqrt{\frac{2\tilde{\varepsilon}_0}{m}}\right]}$$

Бесконачно дугачка нит непроводна са линијском густином λ лежи дуж z-осе.

а) Наћи електростатичко поље у тачки P на растојању x_0 од z-осе

б) У тренутку $t=0$ нит почиње да се креће константном брзином v у позитивном смеру ~~x~~ z осе. Написати израз за запреминску густину струје. Израчунати $A_z(x_0, t)$ за $t > \frac{x_0}{c}$ и за $t < \frac{x_0}{c}$.

в) због цилиндричне симетрије знамо и $A_z(\rho, t)$. Наћи магнетно поље када $t \to \infty$.



а) $\int \vec{E}\, d\vec{S} = \frac{1}{\varepsilon_0}\int \rho\, dV \qquad \vec{E} = E(r_c)\,\vec{e}_{r_c}$

$$E(r_c)\cdot 2 r_c \pi \cdot \cancel{l} = \frac{1}{\varepsilon_0}\lambda \cancel{l} \Rightarrow \boxed{\vec{E}(r_c) = \frac{1}{2\pi\varepsilon_0 r_c}\,\vec{e}_{r_c}}$$

б) $$\rho = \int_{-\infty}^{+\infty} dz'\, \lambda\, \delta(x)\delta(y)\delta(z-z') = \lambda\delta(x)\delta(y)$$

$$\vec{j}(\vec{r}, t) = \lambda v \vec{e}_z\, \delta(x)\delta(y)\eta(t)$$

ретардовани потенцијал

$$\vec{A}(x_0, t) = \frac{\mu_0}{4\pi}\iiint_{-\infty}^{+\infty} \frac{\vec{j}\left(\vec{r}', t - \frac{|\vec{r}-\vec{r}'|}{c}\right)}{|\vec{r}-\vec{r}'|} d\vec{r}' = \frac{\mu_0}{4\pi}\iiint_{-\infty}^{+\infty} \frac{\lambda v \vec{e}_z\, \delta(x')\delta(y')\,\eta\left(t - \frac{1}{c}\sqrt{(x_0-x')^2+y'^2+z'^2}\right)}{\sqrt{(x_0-x')^2+y'^2+z'^2}} d^3 r'$$

$$= \frac{\mu_0}{4\pi}\int_{-\infty}^{+\infty} \frac{\lambda v \vec{e}_z\, \eta\left(t - \frac{1}{c}\sqrt{x_0^2+z'^2}\right)}{\sqrt{x_0^2+z'^2}} dz'$$

$$t > \frac{1}{c}\sqrt{x_0^2+z'^2} \Rightarrow x_0^2 + z'^2 < (ct)^2 \qquad z'^2 < t^2c^2 - x_0^2 \Rightarrow |z'| < \sqrt{t^2c^2-x_0^2} \text{ за } \underline{\underline{x_0 < ct}}$$

1° $t > \frac{x_0}{c} \Rightarrow$ $$A_z(x_0, t) = \frac{\mu_0\lambda v}{4\pi}\int_{-\sqrt{t^2c^2-x_0^2}}^{\sqrt{t^2c^2-x_0^2}} \frac{dz'}{\sqrt{x_0^2+z'^2}} = \frac{\mu_0\lambda v}{4\pi} \ln\left|\frac{z'}{x_0} + \sqrt{1+\left(\frac{z'}{x_0}\right)^2}\right| \Bigg|_{-\sqrt{t^2c^2-x_0^2}}^{\sqrt{t^2c^2-x_0^2}}$$

$$\left(\int \frac{dx}{\sqrt{1+x^2}} = \ln\left|x + \sqrt{1+x^2}\right| + C\right)$$

$$= \frac{\mu_0\lambda v}{4\pi}\ln\left|\frac{\sqrt{t^2c^2-x_0^2} + \sqrt{x_0^2+t^2c^2-x_0^2}}{-\sqrt{t^2c^2-x_0^2} + \sqrt{\cancel{x_0^2}+t^2c^2-\cancel{x_0^2}}}\right| = \frac{\mu_0\lambda v}{4\pi}\ln\left(\frac{1+\sqrt{1-\left(\frac{x_0}{tc}\right)^2}}{1-\sqrt{1-\left(\frac{x_0}{tc}\right)^2}}\right)$$

за $t^2c^2 < x_0^2$ $\quad \eta$ је увек $0 \Rightarrow A_z(x_0, t) = 0$

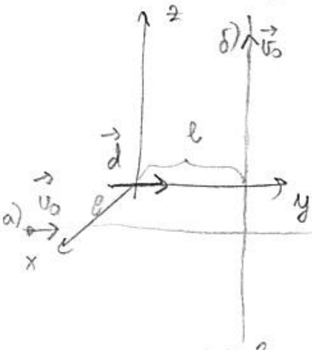
в) $$A_z(r_c, t) = \frac{\mu_0\lambda v}{4\pi}\ln\frac{1+\sqrt{1-\left(\frac{r_c}{ct}\right)^2}}{1-\sqrt{1-\left(\frac{r_c}{ct}\right)^2}}$$

$$\vec{B} = \mathrm{rot}\,\vec{A} = -\frac{\partial A_z(r_c, t)}{\partial r_c}\vec{e}_\varphi = -\vec{e}_\varphi \frac{\mu_0\lambda v}{4\pi}\left(\frac{1}{1+\sqrt{1-\left(\frac{r_c}{ct}\right)^2}}\cdot\frac{-\frac{2r_c}{(ct)^2}}{2\sqrt{1-\left(\frac{r_c}{ct}\right)^2}} + \frac{1}{1-\sqrt{1-\left(\frac{r_c}{ct}\right)^2}}\cdot\frac{-\frac{2r_c}{(ct)^2}}{2\sqrt{1-\left(\frac{r_c}{ct}\right)^2}}\right)$$

$$= +\frac{r_c}{(ct)^2}\frac{\mu_0\lambda v}{4\pi}\vec{e}_\varphi \frac{1}{\sqrt{1-\left(\frac{r_c}{ct}\right)^2}}\cdot\frac{1-\cancel{\sqrt{1-\left(\frac{r_c}{ct}\right)^2}}+1+\cancel{\sqrt{1-\left(\frac{r_c}{ct}\right)^2}}}{\cancel{1}-\cancel{1}+\left(\frac{r_c}{ct}\right)^2} = \frac{r_c}{(ct)^2}\cdot\frac{(ct)^2}{r_c^2}\frac{\mu_0\lambda v}{4\pi}\frac{1}{\sqrt{1-\left(\frac{r_c}{ct}\right)^2}}\cdot 2\vec{e}_\varphi$$

$$= \frac{\mu_0\lambda v}{2\pi r_c}\frac{1}{\sqrt{1-\left(\frac{r_c}{ct}\right)^2}}\vec{e}_\varphi \qquad \text{за } t\to\infty \qquad \boxed{\vec{B} = \frac{\mu_0\lambda v}{2\pi r_c}\vec{e}_\varphi}$$

Електрон масе m и наелектрисања q пролеће на великом растојању l од непокретног дипола $\vec d$. У бесконачности брзина електрона је била $\vec v_0$. Ако је трајекторија честице приближно права линија, израчунати енергију коју електрон израчи ~~у јединици~~ за време кретања ако је а) $\vec d \parallel \vec v_0$; б) $\vec d \perp \vec v_0$.



$\vec d = d\vec e_y$

$$\frac{d\mathcal{E}}{dt} = \frac{1}{6\pi\varepsilon_0 c^3}\,|\ddot{\vec p}(t)|^2$$

$$\vec p = q\vec r \; ; \; \ddot{\vec p} = q\ddot{\vec r}$$

II Њутнов закон $m\ddot{\vec r} = q\vec E$; $\vec E = \frac{1}{4\pi\varepsilon_0}\left(\frac{3(\vec d\vec r)\vec r}{r^5} - \frac{\vec d}{r^3}\right)$ поље дипола

$$\ddot{\vec r} = \frac{q}{m}\vec E = \frac{q}{4\pi\varepsilon_0 m}\left(\frac{3(\vec d\vec r)\vec r}{r^5} - \frac{\vec d}{r^3}\right),$$

$$|\ddot{\vec p}|^2 = \left(\frac{q^2}{4\pi\varepsilon_0 m}\right)^2\left(\frac{3(\vec d\vec r)\vec r}{r^5} - \frac{\vec d}{r^3}\right)^2 = \frac{q^4}{(4\pi\varepsilon_0 m)^2}\left(\frac{9(\vec d\vec r)^2}{r^8} + \frac{d^2}{r^6} - \frac{6(\vec d\vec r)^2}{r^8}\right)$$

$$|\ddot{\vec p}|^2 = \frac{q^4}{(4\pi\varepsilon_0 m)^2}\left(\frac{3(\vec d\vec r)^2}{r^8} + \frac{d^2}{r^6}\right)$$

а) $\vec d \parallel \vec v_0$; $\vec d = d\vec e_y$; $\vec r = l\vec e_x + y\vec e_y$ $\quad \vec d\cdot\vec r = dy$; $r = \sqrt{l^2+y^2}$

$$\frac{d\mathcal{E}}{dt} = \frac{1}{6\pi\varepsilon_0 c^3}\frac{q^4}{(4\pi\varepsilon_0 m)^2}\left(\frac{3d^2y^2}{(l^2+y^2)^4} + \frac{d^2}{(l^2+y^2)^3}\right)$$

поље мало мења брзину честице:

$$\frac{dy}{dt} \approx v_0 \Rightarrow \boxed{dt = \frac{dy}{v_0}}$$

$$d\mathcal{E} = \frac{1}{6\pi\varepsilon_0 c^3}\frac{q^4}{(4\pi\varepsilon_0 m)^2}\left(\frac{3d^2y^2}{(l^2+y^2)^4} + \frac{d^2}{(l^2+y^2)^3}\right)\frac{dy}{v_0}$$

$$\mathcal{E} = \int_{-\infty}^{+\infty}\frac{1}{6\pi\varepsilon_0 c^3 v_0}\frac{q^4}{(4\pi\varepsilon_0 m)^2}\left(\frac{3d^2}{l^5}\frac{\rho^2}{(1+\rho^2)^4} + \frac{d^2}{l^5}\frac{1}{(1+\rho^2)^3}\right)d\rho \; ; \; \rho = \frac{y}{l}$$

$$= \frac{d^2}{6\pi\varepsilon_0 c^3 v_0 l^5}\frac{q^4}{(4\pi\varepsilon_0 m)^2}\int_{-\infty}^{+\infty}\left[3\frac{1}{(1+\rho^2)^3} - 3\frac{1}{(1+\rho^2)^4} + \frac{1}{(1+\rho^2)^3}\right]d\rho = \frac{d^2q^4}{6\pi\varepsilon_0 c^3 v_0 l^5(4\pi\varepsilon_0 m)^2}\int_{-\infty}^{+\infty}\left(\frac{4}{(1+\rho^2)^3} - \frac{3}{(1+\rho^2)^4}\right)d\rho$$

$$I_n = \int_{-\infty}^{+\infty}\frac{d\rho}{(1+\rho^2)^n} \quad \text{смена } \rho = tg\varphi \; ; d\rho = \frac{1}{\cos^2\varphi}d\varphi ; \quad I_n = \int_{-\pi/2}^{\pi/2}\frac{d\varphi}{\cos^2\varphi}\cos^{2n}\varphi = \int_{-\pi/2}^{\pi/2}\cos^{2n-3}\varphi\, d\sin\varphi$$

$$= \underbrace{\cos^{2n-3}\varphi\sin\varphi\Big|_{-\pi/2}^{\pi/2}}_{0}\ _{n\geq 2} - \int_{-\pi/2}^{\pi/2}\sin\varphi\,(2n-3)\cos^{2n-4}\varphi\,(-\sin\varphi)\,d\varphi = (2n-3)\int_{-\pi/2}^{\pi/2}(1-\cos^2\varphi)\cos^{2n-4}\varphi\, d\varphi$$

$$= (2n-3)(I_{n-1} - I_n) \Rightarrow I_n + (2n-3)I_n = (2n-3)I_{n-1} \; ; \; I_n = \frac{2n-3}{2n-2}I_{n-1} \quad n\geq 2$$

$$I_1 = \int_{-\infty}^{+\infty}\frac{d\rho}{(1+\rho^2)} = arctg\rho\Big|_{-\infty}^{+\infty} = \pi \; ; \; I_2 = \frac{1}{2}I_1 = \frac{1}{2}\pi \; ; \; I_3 = \frac{3}{4}I_2 = \frac{3}{8}\pi \; ; \; I_4 = \frac{5}{6}I_3 = \frac{5}{16}\pi$$

$$\mathcal{E} = \frac{d^2q^4}{6\cancel{\pi}\varepsilon_0 c^3 v_0 l^5(4\pi\varepsilon_0 m)^2}\left(4\frac{3}{8}\cancel{\pi} - 3\frac{5}{16}\cancel{\pi}\right) = \frac{d^2q^4}{2\varepsilon_0 c^3 v_0 l^5(4\pi\varepsilon_0 m)^2}\frac{3}{16} = \boxed{\frac{3d^2q^4}{32\varepsilon_0 c^3 v_0 l^5(4\pi\varepsilon_0 m)^2}}$$

б) $\vec d \perp \vec v_0$; $\vec d = d\vec e_y$; $\vec r = l\vec e_y + z\vec e_z$; $\vec d\cdot\vec r = dl$; $\frac{dz}{dt} = v_0$; $\boxed{dt = \frac{dz}{v_0}}$; $r = \sqrt{l^2+z^2}$

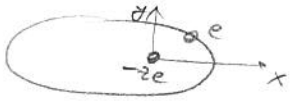
$$d\mathcal{E} = \frac{1}{6\pi\varepsilon_0 c^3}\frac{q^4}{(4\pi\varepsilon_0 m)^2}\left(\frac{3d^2l^2}{(l^2+z^2)^4} + \frac{d^2}{(l^2+z^2)^3}\right)\frac{dz}{v_0}$$

$$\mathcal{E} = \int_{-\infty}^{+\infty}\frac{1}{6\pi\varepsilon_0 c^3}\frac{q^4}{(4\pi\varepsilon_0 m)^2}\left(\frac{3d^2l^2}{l^7(1+\rho^2)^4} + \frac{d^2}{l^5(1+\rho^2)^3}\right)\frac{d\rho}{v_0} = \frac{1}{6\pi\varepsilon_0 c^3}\frac{q^4}{(4\pi\varepsilon_0 m)^2}\frac{d^2}{l^5 v_0}(3I_4 + I_3)$$

$$= \frac{1}{6\cancel{\pi}\varepsilon_0 c^3}\frac{q^4d^2}{l^5 v_0(4\pi\varepsilon_0 m)^2}\left(\frac{15}{16}\cancel{\pi} + \frac{3}{8}\cancel{\pi}\right) = \frac{1}{6\varepsilon_0 c^3 v_0 l^5}\frac{q^4d^2}{(4\pi\varepsilon_0 m)^2}\frac{15+6}{16} = \boxed{\frac{7d^2q^4}{32\varepsilon_0 c^3 v_0 l^5(4\pi\varepsilon_0 m)^2}}$$

Електрон масе m и наелектрисања $q = e < 0$ креће се по елиптичној путањи око наелектрисања $Z|e| = -Ze$. Укупна енергија и ангуларни момент електрона су $\mathcal{E}$ и $\vec{L}$. Одредити енергију коју електрон изгуби на диполно зрачење у току једног периода.

$$dE = \frac{1}{6\pi\varepsilon_0 c^3} |\ddot{\vec{p}}|^2 dt$$



$$\vec{p} = e\vec{r} \; ; \; \ddot{\vec{p}} = e\ddot{\vec{r}} \; ; \; m\ddot{\vec{r}} = \vec{F} = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2}\vec{e}_r = -\frac{k}{r^2}\vec{e}_r \; ; \; \ddot{\vec{p}} = \frac{e}{m}\vec{F} = -\frac{e}{m}\frac{k}{r^2}\vec{e}_r$$

Кретање честице у пољу централне силе $\vec{F} = -\frac{k}{r^2}\vec{e}_r$; $\vec{L} = mr^2\dot{\varphi}\vec{e}_z = \text{const}$.

$$\mathcal{E} = T + U = \frac{m\dot{r}^2}{2} + \frac{mr^2\dot{\varphi}^2}{2} - \frac{k}{r} \qquad \boxed{k = \frac{Ze^2}{4\pi\varepsilon_0}}$$

услов $\dot{r} = 0$ за $\varphi = 0$ $\Rightarrow$ $r = \frac{\tilde{p}}{1 + \tilde{e}\cos\varphi}$ јна трајекторије:

$$\boxed{\tilde{p} = \frac{L^2}{km} \; ; \; \tilde{e} = \sqrt{1 + \frac{2\mathcal{E}L^2}{k^2 m}}}$$ параметри конусног пресека за $\tilde{e} < 1$ елипса, тј. $\mathcal{E} < 0$!

$$dE = \frac{1}{6\pi\varepsilon_0 c^3}\frac{e^2}{m^2}\frac{k^2}{r^4}dt \; ; \quad L = mr^2\frac{d\varphi}{dt} \Rightarrow dt = \frac{mr^2}{L}d\varphi$$

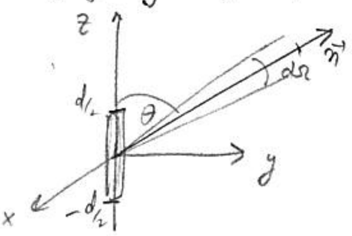
$$dE = \frac{1}{6\pi\varepsilon_0 c^3}\frac{e^2}{m^2}\frac{k^2}{r^4}\frac{mr^2}{L}d\varphi = \frac{e^2k^2 d\varphi}{6\pi\varepsilon_0 c^3 mL r^2}$$

$$E = \frac{e^2k^2}{6\pi\varepsilon_0 c^3 mL}\int_0^{2\pi}\left(\frac{1+\tilde{e}\cos\varphi}{\tilde{p}}\right)^2 d\varphi = \frac{e^2k^2}{6\pi\varepsilon_0 c^3 mL\tilde{p}^2}\int_0^{2\pi}(1 + \tilde{e}^2\underbrace{\cos^2\varphi}_{1/2} + \cancel{2\tilde{e}\cos\varphi}^{\,0})\,d\varphi$$

$$= \frac{e^2k^2}{6\pi\varepsilon_0 c^3 mL\tilde{p}^2}\left(1 + \frac{\tilde{e}^2}{2}\right)2\pi = \frac{e^2k^2}{6\varepsilon_0 c^3 mL}\frac{k^2m^2}{L^4}\left(2 + 1 + \frac{2\mathcal{E}L^2}{k^2 m}\right) = \frac{e^2mk^4}{6\varepsilon_0 c^3L^5}\left(3 + \frac{2\mathcal{E}L^2}{mk^2}\right)$$

$$= \frac{e^2m}{6\varepsilon_0 c^3L^5}\frac{Z^4e^8}{(4\pi\varepsilon_0)^4}\left(3 + \frac{2\mathcal{E}L^2(4\pi\varepsilon_0)^2}{mZ^2e^4}\right) = \boxed{\frac{Z^4e^{10}m}{6\varepsilon_0 c^3L^5(4\pi\varepsilon_0)^4}\left(3 + \frac{2\mathcal{E}L^2(4\pi\varepsilon_0)^2}{Z^2me^4}\right)}$$

Струја у линијској антени дужине d ($d \ll \lambda \ll r$) која је постављена дуж z-осе $(-\frac{d}{2}, \frac{d}{2})$ има облик $I(z,t) = I_0\left(1 - \frac{2|z|}{d}\right)\cos\omega t$. Одредити ангуларну дистрибуцију снаге зрачења и укупну снагу израчену у диполној апроксимацији.



$$dI = \frac{1}{4\pi\varepsilon_0 c^3}\frac{(\ddot{\vec{p}}(\tau)\times\vec{n})^2}{4\pi}d\Omega$$ снага израчена у $d\Omega$

$$\vec{p} = \int \rho\vec{r}\,dV = \int\lambda\vec{r}\,dl$$; немамо λ

јна континуитета; $\frac{\partial\rho}{\partial t} + \operatorname{div}\vec{j} = 0 \Rightarrow \frac{\partial\lambda}{\partial t} + \operatorname{div}\vec{i} = 0$; $\vec{i} = I\vec{e}_z$

$$\frac{\partial\lambda}{\partial t} = -\operatorname{div}\vec{i} = -\frac{\partial I}{\partial z} = -I_0\frac{\partial}{\partial z}\left(1 - \frac{2|z|}{d}\right)\cos\omega t = \boxed{I_0\frac{2}{d}\operatorname{sgn}z\cos\omega t}$$

$$\boxed{\lambda = \frac{2I_0}{d\omega}\operatorname{sgn}z\sin\omega t + f(z)}$$

$$\vec{p} = \int_{-d/2}^{d/2}\lambda z\vec{e}_z\,dz = \frac{2I_0}{d\omega}\int_{-d/2}^{d/2}\operatorname{sgn}z\cdot z\,dz\cdot\sin\omega t\,\vec{e}_z + \underbrace{\int_{-d/2}^{d/2} f(z)\cdot z\,dz}_{A = \text{const.}}\cdot\vec{e}_z$$

$$\vec{p} = \frac{4I_0}{d\omega}\int_0^{d/2} z\,dz\,\sin\omega t\,\vec{e}_z + A\vec{e}_z = \frac{4I_0}{d\omega}\frac{d^2}{8}\sin\omega t\,\vec{e}_z + A\vec{e}_z = \frac{I_0 d}{2\omega}\sin\omega t\,\vec{e}_z + A\vec{e}_z$$

$$\ddot{\vec{p}} = -\frac{I_0 d\omega}{2}\sin\omega t\,\vec{e}_z.$$

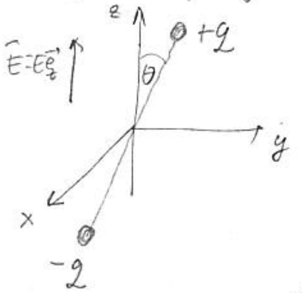
$$dI = \frac{1}{4\pi\varepsilon_0 c^3}\;\frac{I_0^2 d^2\omega^2}{4\cdot 4\pi}\sin^2\omega t\,\left(\vec e_z\times(\sin\theta\,\vec e_r+\cancel{\cos\theta\,\vec e_z})\right)^2 d\Omega = \frac{I_0^2 d^2\omega^2\sin^2\omega t}{64\pi^2\varepsilon_0 c^3}\sin^2\theta\, d\Omega$$

$$\left\langle\frac{dI}{d\Omega}\right\rangle_T = \frac{I_0^2 d^2\omega^2\sin^2\theta}{64\pi^2\varepsilon_0 c^3}\;\frac{1}{T}\int_0^{T=\frac{2\pi}{\omega}}\sin^2\omega t = \frac{I_0^2 d^2\omega^2\sin^2\theta}{64\pi^2\varepsilon_0 c^3}\;\frac{\omega}{2\pi}\int_0^{2\pi/\omega}\frac{1-\cancel{\cos 2\omega t}}{2}\,dt = \frac{I_0^2 d^2\omega^2\sin^2\theta}{128\pi^2 c^3\varepsilon_0}$$

$$\langle I\rangle_T = \frac{I_0^2 d^2\omega^2}{128\pi^2\varepsilon_0 c^3}\int_0^{\pi}\int_0^{2\pi}\sin^2\theta\cdot\underbrace{\sin\theta\, d\theta\, d\varphi}_{d\Omega} = \frac{I_0^2 d^2\omega^2}{128\pi^2\varepsilon_0 c^3}\cdot 2\pi\int_{-1}^{1}(1-\cos^2\theta)\,d\cos\theta$$

$$= \frac{I_0^2 d^2\omega^2}{128\pi^2\varepsilon_0 c^3}\cdot 2\pi\cdot\left(2-\frac{2}{3}\right) = \frac{I_0^2 d^2\omega^2}{64\pi\varepsilon_0 c^3}\;\frac{4}{3} = \boxed{\frac{I_0^2 d^2\omega^2}{48\pi\varepsilon_0 c^3}}$$

Две честице једнаких маса m и наелектрисања $\pm q$ везане су штапом дужине l, занемарљиве масе. Цео систем се налази у сталном електричном пољу $\vec E$ усмереном од негативног ка позитивном наелектрисању. У тренутку $t=0$ штап и поље заклапају мали угао ψ. Одредити интензитет I диполног зрачења овог система.



$$I = \frac{1}{6\pi\varepsilon_0 c^3}|\ddot{\vec p}|^2 \quad ; \quad \vec p = q\frac{l}{2}(\cos\theta\,\vec e_z+\sin\theta\,\vec e_y) - q\frac{l}{2}(-\cos\theta\,\vec e_z-\sin\theta\,\vec e_y)$$

$$\vec p = ql(\cos\theta\,\vec e_z+\sin\theta\,\vec e_y) \qquad \dot{\vec p} = ql(-\sin\theta\,\vec e_z+\cos\theta\,\vec e_y)\cdot\dot\theta$$

$$\ddot{\vec p} = ql(-\cos\theta\,\vec e_z-\sin\theta\,\vec e_y)\cdot\dot\theta^2 + ql(-\sin\theta\,\vec e_z+\cos\theta\,\vec e_y)\cdot\ddot\theta$$

θ − мало $\Rightarrow$ $\boxed{\ddot{\vec p}\approx ql\ddot\theta\,\vec e_y}$ најнижи ред по θ!

$I\cdot\vec\alpha = \vec M$ − јна динамике за штап. $I = 2\cdot m\left(\frac{l}{2}\right)^2 = \frac{ml^2}{2}$

$$\vec\alpha = -\frac{d^2\theta}{dt^2}\cdot\vec e_x = -\ddot\theta\,\vec e_x \;;$$

$$\vec M = \frac{l}{2}(\cancel{\cos\theta\,\vec e_z}+\sin\theta\,\vec e_y)\times qE\,\vec e_z + \frac{l}{2}(-\cancel{\cos\theta\,\vec e_z}-\sin\theta\,\vec e_y)\times(-qE)\,\vec e_z$$

$$\vec M = l\cdot qE\sin\theta\,\vec e_x \approx lqE\theta\,\vec e_x$$

$$\frac{ml^2}{2}\cdot(-\ddot\theta)\,\vec e_x = lqE\theta\,\vec e_x \Rightarrow \ddot\theta + \overbrace{\frac{2lqE}{ml^2}}^{\omega^2}\theta = 0 \qquad \theta = A\cos\omega t + B\sin\omega t\;;\;\omega=\sqrt{\frac{2qE}{ml}}$$

$$\theta(t=0) = \psi = A \;;\; \dot\theta(t=0) = 0 = B \Rightarrow \theta = \psi\cos\left(\sqrt{\frac{2qE}{ml}}\,t\right);\; \boxed{\ddot\theta = -\psi\frac{2qE}{ml}\cos\left(\sqrt{\frac{2qE}{ml}}\,t\right)}$$

$$I = \frac{q^2l^2}{6\pi\varepsilon_0 c^3}\,\psi^2\,\frac{4q^2E^2}{m^2l^2}\cos^2\left(\sqrt{\frac{2qE}{ml}}\,t\right) = \boxed{\frac{2q^4E^2\psi^2}{3\pi\varepsilon_0 c^3 m^2}\cos^2\left(\sqrt{\frac{2qE}{ml}}\,t\right)}$$

Нерелативистички електрон креће се у сталном константном и хомогеном електричном пољу $\vec{E}=E\vec{e}_z$. Наћи интензитет магнетног диполног зрачења електрона, ако је у почетном тренутку електрон био у координатном почетку и имао брзину $\vec{v}_0=v_0\vec{e}_x$.

$$I=\frac{1}{6\pi\varepsilon_0 c^5}\,|\ddot{\vec{m}}(\tau)|^2 \qquad \tau=t-\frac{r}{c}$$

$\vec{m}=\frac{1}{2}\int \vec{r}\times\vec{j}\,dV=\frac{1}{2}\,\vec{r}\times e\vec{v}$; $\vec{r}(t)$ јна кретања електрона; $\vec{v}(t)$ - брзина електрона

$\dot{\vec{m}}=\cancel{\frac{1}{2}\vec{v}\times e\vec{v}}^{\,0}+\frac{1}{2}\vec{r}\times e\vec{a}=\frac{e}{2}\vec{r}\times\vec{a}$; II Њутнов закон $m\vec{a}=e\vec{E}$; $\vec{a}=\frac{e\vec{E}}{m}$

$\dot{\vec{m}}=\frac{e}{2}\vec{r}\times\frac{e\vec{E}}{m}=\frac{e^2}{2m}\vec{r}\times\vec{E}$; $\boxed{\ddot{\vec{m}}=\frac{e^2}{2m}\vec{v}\times\vec{E}}$ треба нам $\vec{v}(t)$!

$m\vec{a}=e\vec{E}=eE\vec{e}_z$; $m\frac{dv_x}{dt}=0 \Rightarrow \boxed{v_x=v_0}$; $m\frac{dv_y}{dt}=0\Rightarrow\boxed{v_y=0}$

$m\frac{dv_z}{dt}=eE$; $\boxed{v_z=\frac{eE}{m}t}$

$\boxed{\vec{v}(t)=v_0\vec{e}_x+\frac{e\vec{E}}{m}t}$ $\qquad \ddot{\vec{m}}=\frac{e^2}{2m}\left(\vec{v}_0+\cancel{\frac{e\vec{E}}{m}t}\right)\times\vec{E}=\boxed{\frac{e^2}{2m}\vec{v}_0\times\vec{E}}$

$$I=\frac{1}{6\pi\varepsilon_0 c^5}\frac{e^4}{4m^2}\left(\vec{v}_0\times\vec{E}\right)^2=\boxed{\frac{e^4 v_0^2E^2}{24\pi\varepsilon_0 c^5 m^2}}$$

За линеарни хармонијски осцилатор масе m и наелектрисања q и угаоне фреквенције ω израчунати угаону расподелу интензитета квадруполног зрачења.

$\frac{dI}{d\Omega}=\frac{\mu_0}{(4\pi)^2 36c^3}\left[\left(\dddot{D}\vec{n}\right)\times\vec{n}\right]^2$; $\vec{n}=\vec{e}_r$

ЛХО $\vec{r}=a\cos(\omega\tau)\,\vec{e}_z$;

$D_{xx}=q\left(3\cancel{x^2}-a^2\cos^2(\omega\tau)\right)=-qa^2\cos^2(\omega\tau)$; $D_{yy}=q\left(3\cancel{y^2}-a^2\cos^2(\omega\tau)\right)=-qa^2\cos^2(\omega\tau)$

$D_{zz}=2qa^2\cos^2\omega\tau$; $D_{xy}=D_{xz}=D_{yz}=0$

$$\hat{D}=qa^2\cos^2(\omega\tau)\begin{bmatrix}-1&&\\&-1&\\&&2\end{bmatrix}=qa^2\frac{1+\cos(2\omega\tau)}{2}\begin{bmatrix}-1&&\\&-1&\\&&2\end{bmatrix}$$

$$\dddot{\hat{D}}=\frac{qa^2}{2}(2\omega)^3\sin(2\omega\tau)\begin{bmatrix}-1&&\\&-1&\\&&2\end{bmatrix}=4qa^2\omega^3\sin(2\omega\tau)\begin{bmatrix}-1&&\\&-1&\\&&2\end{bmatrix}$$

$\vec{e}_r=\begin{pmatrix}\sin\theta\cos\varphi\\ \sin\theta\sin\varphi\\ \cos\theta\end{pmatrix}$ $\qquad \left(\dddot{\hat{D}}\vec{e}_r\right)=4qa^2\omega^3\sin(2\omega\tau)\begin{pmatrix}-\sin\theta\cos\varphi\\ -\sin\theta\sin\varphi\\ 2\cos\theta\end{pmatrix}$

$\left(\dddot{\hat{D}}\vec{e}_r\right)\times\vec{e}_r=4qa^2\omega^3\sin(2\omega\tau)\left(-\sin\theta\,\vec{e}_\rho+2\cos\theta\,\vec{e}_z\right)\times\left(\sin\theta\,\vec{e}_\rho+\cos\theta\,\vec{e}_z\right)$

$=4qa^2\omega^3\sin(2\omega\tau)\left(2\sin\theta\cos\theta\,\vec{e}_\varphi+\sin\theta\cos\theta\,\vec{e}_\varphi\right)=12qa^2\omega^3\sin(2\omega\tau)\frac{1}{2}\sin(2\theta)\,\vec{e}_\varphi$

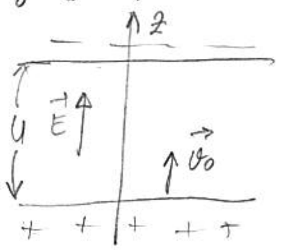
$\frac{dI}{d\Omega}=\frac{\mu_0}{(4\pi)^2\cancel{36}c^3}\cancel{36}\,q^2a^4\omega^6\sin^2(2\omega\tau)\sin^2(2\theta)=\frac{\mu_0 q^2a^4\omega^6\sin^2(2\omega\tau)}{(4\pi)^2c^3}\sin^2(2\theta)$

$I=\frac{\mu_0 q^2a^4\omega^6\sin^2(2\omega\tau)}{(4\pi)^2c^3}\int_0^{\pi}\int_0^{2\pi}\underbrace{\sin\theta\,d\theta\,d\varphi}_{d\Omega}\cdot\sin^2(2\theta)=\frac{\mu_0 q^2a^4\omega^6\sin^2(2\omega\tau)}{(4\pi)^{\cancel{2}}c^3}2\pi\cdot4\int_{-1}^{1}(1-\cos^2\theta)\cos^2\theta\,d\cos\theta$

$=\frac{\mu_0 q^2a^4\omega^6\sin^2(2\omega\tau)}{4\pi c^3}\,2\cdot2\left(\frac{1}{3}-\frac{1}{5}\right)=\boxed{\frac{2\mu_0 q^2a^4\omega^6\sin^2(2\omega\tau)}{15\pi c^3}}$ Укупна снага

Зрачење релативистичке честице

Растојање између плоча кондензатора је l, а напон на кондензатору је U. Нормално на плоче у смеру вектора електричног поља $\vec E$ креће се протон масе m и наелектрисања e. Почетна брзина протона је v_0 и упоредиво је са брзином светлости. Одредити енергију коју протон зрачи за време кретања кроз кондензатор.



$$\vec E = \frac{U}{l}\vec e_z$$

$$d\mathcal{E}_{изр} = \frac{1}{6\pi\varepsilon_0 c^3}\frac{e^2}{m^2}\frac{\vec F^2 - \frac{1}{c^2}(\vec v\cdot\vec F)^2}{1-v^2/c^2}\,dt$$

$$\vec F = e\vec E\,;\quad \vec F^2 = e^2E^2\,;\quad (\vec F\cdot\vec v)^2 = e^2E^2v^2 \qquad \vec v = v\vec e_z$$

$$\frac{d\vec p}{dt} = \vec F \qquad \frac{dp_x}{dt} = \frac{dp_y}{dt} = 0 \Rightarrow v_x = v_y = 0\,;\quad \frac{dp_z}{dt} = eE\,;\ \boxed{p_z = p_0 + eEt}$$

$$\frac{d\mathcal E}{dt} = \vec F\cdot\vec v = eEv = eE\frac{dz}{dt} \Rightarrow \boxed{\mathcal E = \mathcal E_0 + eEz}$$ ← зависност енергије протона од z.

$$d\mathcal E_{изр} = \frac{1}{6\pi\varepsilon_0 c^3}\frac{e^2}{m^2}\frac{e^2E^2 - \frac{1}{c^2}e^2E^2v^2}{1-v^2/c^2}dt = \frac{1}{6\pi\varepsilon_0 c^3}\frac{e^2}{m^2}\frac{e^2E^2\cancel{(1-v^2/c^2)}}{\cancel{1-v^2/c^2}}dt = \frac{e^4E^2}{6\pi\varepsilon_0c^3m^2}dt$$

$$\boxed{\mathcal E_{изр} = \frac{e^4E^2}{6\pi\varepsilon_0c^3m^2}t_*}$$

t_* - време за које стигне до горње плоче.

$$\mathcal E(l) = \mathcal E_0 + eEl = \mathcal E_0 + e\frac{U}{l}l = \mathcal E_0 + eU\,;\quad \mathcal E(l)^2 = m^2c^4 + c^2p^2(t_*) \qquad p(t_*) = p_0 + eEt_*$$

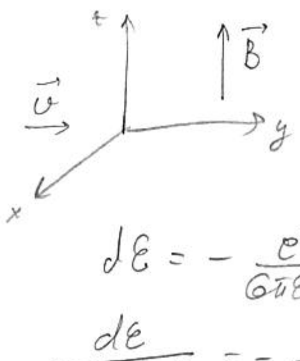
$$\mathcal E(l)^2 = (\mathcal E_0 + eU)^2 = m^2c^4 + c^2(p_{0z} + eEt_*)^2 \Rightarrow t_* = \left(\sqrt{\frac{(\mathcal E_0+eU)^2 - m^2c^4}{c^2}} - p_{0z}\right)\frac{1}{eE}$$

$$p_{0z} = \frac{mv_0}{\sqrt{1-v_0^2/c^2}}\,;\qquad \mathcal E_0 = \frac{mc^2}{\sqrt{1-v_0^2/c^2}}$$

$$\mathcal E_{изр} = \frac{e^{\cancel{4}\,3}U^{\cancel{2}}}{6\pi\varepsilon_0c^3m^2l^{\cancel{2}}}\frac{\cancel{l}}{e\cancel{U}}\left(\sqrt{\frac{(\mathcal E_0+eU)^2-m^2c^4}{c^2}} - p_0\right) = \boxed{\frac{e^3U}{6\pi\varepsilon_0c^3m^2l}\left(\sqrt{\frac{(\mathcal E_0+eU)^2-m^2c^4}{c^2}} - p_0\right)}$$

Нормално на хомогено магнетно поље $\vec B$ креће се релативистички електрон масе m и наелектрисања e. У почетном тренутку енергија електрона је била $\mathcal E_0$. Одредити закон зрачења енергије електрона и наћи његов нерелатив. лимес.

$$d\mathcal E_{изр} = \frac{1}{6\pi\varepsilon_0c^3}\frac{e^2}{m^2}\frac{|\vec F|^2 - \frac{1}{c^2}(\vec F\cdot\vec v)^2}{1-v^2/c^2}dt \qquad d\mathcal E_{изр} = -d\mathcal E \qquad \mathcal E \text{ - енергија електрона}$$



$$|\vec F| = |e\vec v\times\vec B| = evB\,;\quad \vec v\perp\vec B\,;\quad \vec F\cdot\vec v = e(\vec v\times\vec B)\cdot\vec v = 0$$

$$-d\mathcal E = \frac{e^2}{6\pi\varepsilon_0c^3m^2}\frac{e^2v^2B^2}{1-v^2/c^2}dt\,;\qquad \mathcal E = \frac{mc^2}{\sqrt{1-v^2/c^2}}$$

$$1 - v^2/c^2 = \left(\frac{mc^2}{\mathcal E}\right)^2 \qquad v^2 = c^2\left(1 - \frac{m^2c^4}{\mathcal E^2}\right)$$ (хоћу v преко $\mathcal E$)

$$d\mathcal E = -\frac{e^4B^2}{6\pi\varepsilon_0c^3m^2}\frac{c^2\left(1-\frac{m^2c^4}{\mathcal E^2}\right)}{\frac{m^2c^4}{\mathcal E^2}}dt = -\frac{e^4B^2}{6\pi\varepsilon_0c^5m^4}(\mathcal E^2 - m^2c^4)dt$$

$$\frac{d\mathcal E}{\mathcal E^2 - m^2c^4} = -\frac{e^4B^2}{6\pi\varepsilon_0c^5m^4}dt$$ раздвајамо променљиве

$$\frac{d\varepsilon}{\varepsilon^2-m^2c^4} = d\varepsilon\left(\frac{1}{\varepsilon-mc^2}-\frac{1}{\varepsilon+mc^2}\right)\frac{1}{2mc^2} = -\frac{e^4B^2}{6\pi\varepsilon_0 c^5 m^4}\,dt$$

$$\int d\varepsilon\left(\frac{1}{\varepsilon-mc^2}-\frac{1}{\varepsilon+mc^2}\right) = -\int\frac{e^4B^2}{3\pi\varepsilon_0 c^3 m^3}\,dt$$

$$\ln\frac{\varepsilon-mc^2}{\varepsilon_0-mc^2}-\ln\frac{\varepsilon+mc^2}{\varepsilon_0+mc^2} = -\overbrace{\frac{e^4B^2}{3\pi\varepsilon_0 c^3 m^3}}^{\alpha}t$$

ε_0 = почетна енергија електрона.

$$\frac{\varepsilon-mc^2}{\varepsilon+mc^2}\cdot\frac{\varepsilon_0+mc^2}{\varepsilon_0-mc^2} = e^{-\alpha t}\ ;\quad (\varepsilon-mc^2)(\varepsilon_0+mc^2)=(\varepsilon+mc^2)(\varepsilon_0-mc^2)e^{-\alpha t}$$

$$\varepsilon\left(\varepsilon_0+mc^2-(\varepsilon_0-mc^2)e^{-\alpha t}\right) = mc^2\left(\varepsilon_0+mc^2+(\varepsilon_0-mc^2)e^{-\alpha t}\right)$$

$$\boxed{\varepsilon(t) = mc^2\,\frac{\varepsilon_0+mc^2+(\varepsilon_0-mc^2)e^{-\alpha t}}{\varepsilon_0+mc^2-(\varepsilon_0-mc^2)e^{-\alpha t}}} = \boxed{mc^2\,\frac{1+\frac{\varepsilon_0-mc^2}{\varepsilon_0+mc^2}e^{-\alpha t}}{1-\frac{\varepsilon_0-mc^2}{\varepsilon_0+mc^2}e^{-\alpha t}}}$$

нерелативистички лимес: $\varepsilon_0 = \frac{mc^2}{\sqrt{1-v_0^2/c^2}} \simeq mc^2\left(1+\frac{v_0^2}{2c^2}+\dots\right)$

$$\frac{\varepsilon_0-mc^2}{\varepsilon_0+mc^2} \simeq \frac{\cancel{mc^2}\left(1+\frac{v_0^2}{2c^2}+\dots-1\right)}{\cancel{mc^2}\left(1+\frac{v_0^2}{2c^2}+\dots+1\right)} \simeq \frac{v_0^2}{4c^2}$$ водећи члан!

$$\varepsilon(t) = mc^2\,\frac{1+\frac{v_0^2}{4c^2}e^{-\alpha t}}{1-\frac{v_0^2}{4c^2}e^{-\alpha t}} \simeq mc^2\left(1+\frac{v_0^2}{4c^2}e^{-\alpha t}\right)\left(1+\frac{v_0^2}{4c^2}e^{-\alpha t}\right) \simeq mc^2\left(1+\frac{v_0^2}{2c^2}e^{-\alpha t}\dots\right)$$

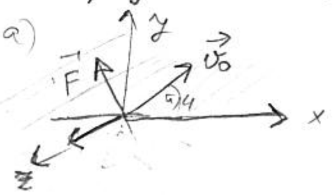
$$\boxed{\varepsilon(t) \simeq mc^2+\frac{mv_0^2}{2}e^{-\alpha t}}$$

Релативистичка честица масе m и наелектрисања $q=e$ улети у простор у коме постоји константно и хомогено магнетно поље $\vec{B}$, а са граничном равни заклапа угао од $\pi/4$. Одредити енергију коју честица израчи за време кретања у делу простора у коме постоји $\vec{B}$, ако је: $(\vec{v}_0 \perp \vec{B})$

а) у поч. тренутку Лоренцова сила усмерена ка полупростору у коме постоји поље

б) у поч. тренутку Лоренцова сила усмерена ка полупростору у коме нема поља.

а)



у делу простора $y>0$ постоји магнетно поље $\vec{B}=B\vec{e}_z$

$\vec{F}=q\vec{v}\times\vec{B}$ $\qquad q=e<0$!

$$\vec{F} = e\begin{vmatrix}\vec{e}_x & \vec{e}_y & \vec{e}_z\\ v_x & v_y & 0\\ 0 & 0 & B\end{vmatrix} = eB\left(v_y\vec{e}_x - v_x\vec{e}_y\right);$$

$$d\varepsilon_{изр} = \frac{e^2}{6\pi\varepsilon_0 c^3 m^2}\,\frac{\vec{F}^2-\frac{1}{c^2}(\vec{F}\cdot\vec{v})^2}{1-v^2/c^2}\,dt\ ;\quad \vec{F}\cdot\vec{v}=0\ ;\quad F^2=e^2B^2v^2$$

$$d\varepsilon_{изр} = \frac{e^2}{6\pi\varepsilon_0 c^3 m^2}\,\frac{e^2B^2v^2}{1-v^2/c^2}\,dt\ ;\quad \frac{d\vec{p}}{dt}=\vec{F}\ ;\quad \frac{d\varepsilon}{dt}=0\quad \varepsilon=\varepsilon_0 \Rightarrow v=v_0=const.$$

(занем. кочење зрачењем)

$$\varepsilon_{изр} = \frac{e^4B^2v_0^2}{6\pi\varepsilon_0 c^3 m^2(1-v_0^2/c^2)}\,t_*\ ;$$ t_* - време које честица проведе у области са пољем.

$$\frac{dp_x}{dt} = eB\frac{dy}{dt} \Rightarrow p_x = eBy+p_{0x}\ ;\quad \frac{dp_y}{dt} = -eB\frac{dx}{dt} \Rightarrow p_y = -eBx+p_{0y}$$

$$p_x = \frac{m v_x}{\sqrt{1-v^2/c^2}} = \frac{m\frac{dx}{dt}}{\sqrt{1-v^2/c^2}} = \frac{m\,dx}{d\tau} = eBy + p_{0x}; \qquad p_y = \frac{m\,dy}{d\tau} = -eBx + p_{0y}$$

$$\frac{m\,d^2x}{d\tau^2} = eB\frac{dy}{d\tau} = eB\left(-\frac{eB}{m}x + \frac{p_{0y}}{m}\right); \qquad \frac{d^2x}{d\tau^2} + \left(\frac{eB}{m}\right)^2 x = \frac{eB\,p_{0y}}{m^2}$$

$$d\tau = dt\sqrt{1-v^2/c^2} = dt\sqrt{1-v_0^2/c^2} \Rightarrow \boxed{\tau = t\sqrt{1-v_0^2/c^2}} \qquad \boxed{\omega = \frac{eB}{m}}$$

$$x(\tau) = A\cos\omega\tau + B\sin\omega\tau + \frac{p_{0y}}{eB}; \qquad y(\tau) = \underbrace{\frac{1}{eB}m\omega}_{=1}(-\sin\omega\tau\cdot A + B\cos\omega\tau) - \frac{p_{0x}}{eB}$$

$$y(\tau) = -A\sin\omega\tau + B\cos\omega\tau - \frac{p_{0y}}{eB}$$

$$x(\tau=0) = 0 = A + \frac{p_{0y}}{eB} \qquad y(\tau=0) = 0 = B - \frac{p_{0x}}{eB} \Rightarrow A = -\frac{p_{0y}}{eB}; \; B = \frac{p_{0x}}{eB}$$

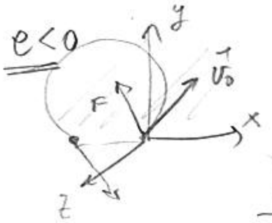
$$x(\tau) = +\frac{p_{0y}}{eB}(1-\cos\omega\tau) + \frac{p_{0x}}{eB}\sin\omega\tau; \qquad y(\tau) = \frac{p_{0x}}{eB}(\cos\omega\tau - 1) + \frac{p_{0y}}{eB}\sin\omega\tau$$

$$\vec{p}_0 = \frac{m\left(\frac{v_0}{\sqrt2}\vec{e}_x + \frac{v_0}{\sqrt2}\vec{e}_y\right)}{\sqrt{1-v_0^2/c^2}}; \qquad \boxed{\begin{aligned} x(\tau) &= \frac{m v_0}{eB\sqrt2\sqrt{1-v_0^2/c^2}}(1-\cos\omega\tau + \sin\omega\tau) \\ y(\tau) &= \frac{m v_0}{eB\sqrt2\sqrt{1-v_0^2/c^2}}(\cos\omega\tau + \sin\omega\tau - 1) \end{aligned}}$$

$$p_x(\tau) = \frac{m\,dx}{d\tau} = \frac{m^2 v_0}{eB\sqrt2\sqrt{1-v_0^2/c^2}}\,\omega(\sin\omega\tau + \cos\omega\tau) = \frac{m v_0}{\sqrt2\sqrt{1-v_0^2/c^2}}(\sin\omega\tau + \cos\omega\tau)$$

$$p_y(\tau) = \frac{m\,dy}{d\tau} = \frac{m^2 v_0}{eB\sqrt2\sqrt{1-v_0^2/c^2}}(-\sin\omega\tau + \cos\omega\tau) = \frac{m v_0}{\sqrt2\sqrt{1-v_0^2/c^2}}(-\sin\omega\tau + \cos\omega\tau)$$

а) $e<0$



На излазу из простора са $\vec{B}$

$$p_x(\tau_*) = p_x(0) \qquad p_y(\tau_*) = -p_y(0)$$

$$\frac{m v_0}{\sqrt2\sqrt{1-v_0^2/c^2}}(\sin\omega\tau_* + \cos\omega\tau_*) = \frac{m v_0}{\sqrt2\sqrt{1-v_0^2/c^2}} \Rightarrow \sin\omega\tau_* + \cos\omega\tau_* = 1$$

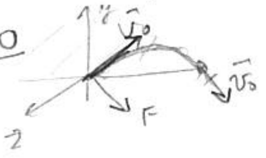
$$\frac{m v_0}{\sqrt2\sqrt{1-v_0^2/c^2}}(-\sin\omega\tau_* + \cos\omega\tau_*) = -\frac{m v_0}{\sqrt2\sqrt{1-v_0^2/c^2}} \Rightarrow -\sin\omega\tau_* + \cos\omega\tau_* = -1$$

$$\left.\right\} \cos\omega\tau_* = 0, \; \sin\omega\tau_* = 1$$

$$\omega = \frac{eB}{m} < 0 \Rightarrow \cos(-|\omega|\tau_*) = 0; \; \sin(-|\omega|\tau_*) = 1; \; \sin(|\omega|\tau_*) = -1 \Rightarrow |\omega|\tau_* = \frac{3\pi}{2}; \; \tau_* = \frac{3\pi}{2|\omega|}$$

$$t_* = \frac{\tau_*}{\sqrt{1-v_0^2/c^2}} = \frac{-3\pi\,m}{2\sqrt{1-v_0^2/c^2}\,eB}; \qquad \mathcal{E}_{изр} = \frac{-e^4B^2v_0^2}{6\pi\varepsilon_0 c^3 m^2(1-v_0^2/c^2)}\,\frac{3\pi m}{2\sqrt{1-v_0^2/c^2}\,eB} = \boxed{-\frac{e^3 B v_0^2}{4\varepsilon_0 c^3 m (1-v_0^2/c^2)^{3/2}}}$$

б) $e>0$



на излазу $p_x(\tau_*) = p_x(0)$; $p_y(\tau_*) = -p_y(0)$; $\omega > 0$ $\boxed{\cos\omega\tau_* = 0, \; \sin\omega\tau_* = 1}$

$$\Rightarrow \omega\tau_* = \frac{\pi}{2} \qquad t_* = \frac{\pi}{2\sqrt{1-v_0^2/c^2}}; \qquad \boxed{\mathcal{E}_{изр} = \frac{e^3 B v_0^2}{12\varepsilon_0 c^3 m (1-v_0^2/c^2)^{3/2}}}$$