EM таласи у вакууму

$$\Delta\vec{E}-\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}=0 \qquad \Delta\vec{B}-\frac{1}{c^2}\frac{\partial^2\vec{B}}{\partial t^2}=0$$ таласне j-не

$$f(\vec r,t)=\frac{1}{(2\pi)^4}\int d\omega\, d^3\vec{k}\; f_{\vec k,\omega}\, e^{i\omega t-i\vec k\vec r} \rightsquigarrow f_{\vec k,\omega}=\int_{-\infty}^{+\infty} dt'\, d^3\vec r\,'\, f(\vec r\,',t')\, e^{-i\omega t'+i\vec k\vec r\,'}$$

развој по равним и монохром. таласима!

$$f(\vec r,t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega\, f_\omega(\vec r)\, e^{i\omega t} \qquad f_\omega(\vec r)=\int dt'\, f(\vec r,t')\, e^{-i\omega t'}$$ развој по монохром. тал.

$$f(\vec r,t)=\frac{1}{(2\pi)^3}\int_{-\infty}^{+\infty} d^3\vec k\, f_{\vec k}(t)\, e^{-i\vec k\vec r}, \quad f_{\vec k}(t)=\int d^3\vec r\,'\, f(\vec r\,',t)\, e^{i\vec k\vec r\,'}$$ развој по равним таласима

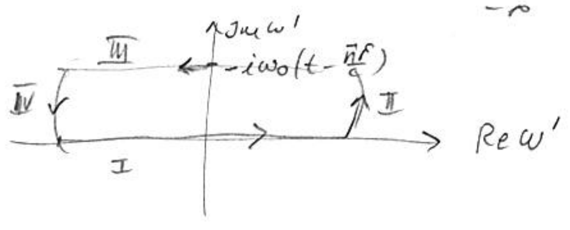
Фурије компонента електричног поља је $\vec E(\vec k,\omega)=\frac{(2\pi)^4}{\omega_0}\vec E_0\,\delta(\vec k-\frac{\omega\vec n}{c})\,e^{-\frac{\omega^2}{\omega_0^2}}$, одредити $\vec E(\vec r,t)$.

$$\vec E(\vec r,t)=\frac{1}{(2\pi)^4}\int d\omega\, d^3\vec k\; E_{\vec k,\omega}\, e^{i\omega t-i\vec k\vec r}=\cancel{\frac{1}{(2\pi)^4}}\int d\omega\, d^3\vec k\, \cancel{\frac{(2\pi)^4}{\omega_0}}\vec E_0\,\delta(\vec k-\frac{\omega\vec n}{c})\,e^{-\frac{\omega^2}{\omega_0^2}}e^{i\omega t-i\vec k\vec r}$$

$$=\frac{\vec E_0}{\omega_0}\int_{-\infty}^{+\infty} d\omega\, e^{i\omega t-i\frac{\omega\vec n}{c}\vec r-\frac{\omega^2}{\omega_0^2}}=\frac{\vec E_0}{\omega_0}\int_{-\infty}^{+\infty} d\omega\, e^{-\left(\frac{\omega}{\omega_0}-\frac{i\omega_0}{2}(t-\frac{\vec n\vec r}{c})\right)^2-\frac{\omega_0^2}{4}(t-\frac{\vec n\vec r}{c})^2}$$

смена $\frac{\omega}{\omega_0}-i\omega_0(t-\frac{\vec n\vec r}{c})=\omega'$

$$\vec E(\vec r,t)=\vec E_0\, e^{-\frac{\omega_0^2}{4}(t-\frac{\vec n\vec r}{c})^2}\int_{-\infty}^{+\infty} d\omega'\, e^{-\omega'^2}=\vec E_0\sqrt{\pi}\, e^{-\frac{\omega_0^2}{4}(t-\frac{\vec n\vec r}{c})^2}$$



$$\oint f(z)\,dz=2\pi i\,\mathrm{Res}\, f(z)=0$$
$$\Rightarrow \int_{III} f(z)\,dz=\int_{I} f(z)\,dz$$

Електрично поље је описано сферно сим. потенцијалом $V=q\frac{e^{-\frac{r}{a}}}{r}$. Одредити $\vec E$ развијено по равним таласима.

$$-\nabla V=\vec E(\vec r)=\frac{1}{(2\pi)^3}\int d^3\vec k\; \vec E_{\vec k}(\cancel{t})\, e^{-i\vec k\vec r}\;;\quad \vec E_{\vec k}(\cancel{t})=\int d^3\vec r\; \vec E(\vec r)\, e^{i\vec k\vec r}$$

јер V не зависи од t!

$$\left(\vec E_{\vec k}=\int d^3\vec r\; \vec E(\vec r)\, e^{i\vec k\vec r}=\int d^3\vec r\,(-\nabla V(\vec r))\, e^{i\vec k\vec r}\right)$$

$$V(r)=\frac{1}{(2\pi)^3}\int d^3\vec k\; V_{\vec k}\, e^{-i\vec k\vec r}\;;\quad \nabla V(\vec r)=\frac{1}{(2\pi)^3}\int d^3\vec k\; V_{\vec k}(-i\vec k)\, e^{-i\vec k\vec r}=-\vec E(\vec r)$$

$$\Rightarrow \boxed{E_{\vec k}=i\vec k V_{\vec k}}$$ нађемо $V_{\vec k}$!

$$V_{\vec k} = \int d^3\vec r\, V(\vec r)\, e^{i\vec k\vec r} = \int d^3\vec r\, q\,\frac{e^{-r/a}}{r}\, e^{i\vec k\vec r} \quad ; \quad \vec k = k\vec e_z \text{ сферне коорд.}$$

$$= \int_0^\infty\int_0^\pi\int_0^{2\pi} q\,\frac{e^{-r/a}}{r}\, e^{ikr\cos\theta}\, r^2\sin\theta\, dr\, d\theta\, d\varphi = -2\pi q\int \frac{e^{-r/a}}{r}\, r^2 dr \int_0^\pi e^{ikr\cos\theta}\, d\cos\theta$$

$$= -2\pi q\int_0^\infty dr\, e^{-r/a}\cdot \cancel{r}\,\frac{1}{ik\cancel{r}}\, e^{ikr\cos\theta}\Big|_0^\pi = -\frac{2\pi q}{ik}\int_0^\infty dr\, e^{-\frac{r}{a}}\left(e^{-ikr}-e^{ikr}\right)$$

Важи $\int e^{-\alpha x}dx = -\frac{1}{\alpha}e^{-\alpha x}+C$ и за $\alpha\in\mathbb{C}$ продужн.

$$= -\frac{2\pi q}{ik}\left(\int_0^\infty dr\, e^{-r(\frac1a+ik)} - \int_0^\infty dr\, e^{-r(\frac1a-ik)}\right) = -\frac{2\pi q}{ik}\left(-\frac{1}{\frac1a+ik}e^{-r(\frac1a+ik)} + \frac{1}{\frac1a-ik}e^{-r(\frac1a-ik)}\right)\Big|_0^\infty$$

$$= -\frac{2\pi q}{ik}\left(\frac{1}{\frac1a+ik}-\frac{1}{\frac1a-ik}\right) = \frac{2\pi i q a}{k}\,\frac{\cancel{1}-ika-\cancel{1}-ika}{1+k^2a^2} = \frac{2\pi i q a}{1+k^2a^2}\cdot(-2ia) = \boxed{\frac{4\pi q a^2}{1+k^2a^2}}$$

може и без компл. интеграла;

$$V_{\vec k} = -\frac{2\pi q}{ik}\int_0^\infty dr\, e^{-\frac{r}{a}}\underbrace{\left(e^{-ikr}-e^{ikr}\right)}_{-2i\sin kr} = \frac{4\pi q}{k}\int_0^\infty e^{-\frac{r}{a}}\sin kr\, dr$$

$$\int e^{-\alpha x}\sin\beta x\, dx = -\frac{1}{\alpha}\int \sin\beta x\, de^{-\alpha x} = -\frac{1}{\alpha}\sin\beta x\, e^{-\alpha x} + \frac{1}{\alpha}\int e^{-\alpha x}\beta\cdot\cos\beta x\, dx$$

$$= -\frac{1}{\alpha}\sin\beta x\, e^{-\alpha x} - \frac{\beta}{\alpha^2}\int\cos\beta x\, de^{-\alpha x} = -\frac{1}{\alpha}\sin\beta x\, e^{-\alpha x} - \frac{\beta}{\alpha^2}\cos\beta x\, e^{-\alpha x}$$

$$+\frac{\beta}{\alpha^2}\int e^{-\alpha x}(-\beta)\sin\beta x\, dx$$

$$\left(1+\frac{\beta^2}{\alpha^2}\right)\int e^{-\alpha x}\sin\beta x\, dx = -\frac{1}{\alpha^2}\left(\alpha\sin\beta x+\beta\cos\beta x\right)e^{-\alpha x} + \text{const.}$$

$$\int e^{-\alpha x}\sin\beta x\, dx = -\frac{1}{\alpha^2+\beta^2}\left(\alpha\sin\beta x+\beta\cos\beta x\right)e^{-\alpha x} + \text{const.}$$

$$\Rightarrow V_{\vec k} = -\frac{4\pi q}{k}\,\frac{1}{\frac{1}{a^2}+k^2}\left(\frac1a\sin kr + k\cos kr\right)e^{-\frac{r}{a}}\Big|_0^\infty = +\frac{4\pi q}{k}\,\frac{1}{\frac{1}{a^2}+k^2}\,k = \boxed{\frac{4\pi q a^2}{1+k^2a^2}}$$

$$\boxed{E_{\vec k} = i\vec k\, V_{\vec k} = \frac{i4\pi q a^2\vec k}{1+k^2a^2}}$$

$$\boxed{\vec E(\vec r) = \frac{1}{(2\pi)^3}\int d^3\vec k\, \frac{i4\pi q a^2\vec k}{1+k^2a^2}\, e^{-i\vec k\vec r}}$$

Одредити скаларни и векторски потенцијал тачкастог наелектрисања q које се креће константном брзином $\vec{v}$ користећи се развојем на равне монохроматске таласе.

Решавамо: $\Delta\varphi - \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} = -\frac{1}{\varepsilon_0}\rho$

развијемо φ и ρ

$$\varphi = \frac{1}{(2\pi)^4}\int d\omega\, d^3\vec{k}\; \varphi_{\vec{k},\omega}\, e^{i\omega t - i\vec{k}\vec{r}} \quad ; \quad \rho = \frac{1}{(2\pi)^4}\int d\omega\, d^3\vec{k}\; \rho_{\vec{k},\omega}\, e^{i\omega t - i\vec{k}\vec{r}}$$

$$\Delta\varphi = \frac{1}{(2\pi)^4}\int d\omega\, d^3\vec{k}\; (-k^2)\,\varphi_{\vec{k},\omega}\, e^{i\omega t - i\vec{k}\vec{r}} \quad ; \quad \frac{\partial^2\varphi}{\partial t^2} = \frac{1}{(2\pi)^4}\int d\omega\, d^3\vec{k}\; (-\omega^2)\,\varphi_{\vec{k},\omega}\, e^{i\omega t - i\vec{k}\vec{r}}$$

$$\frac{1}{(2\pi)^4}\int d\omega\, d^3\vec{k}\left(-k^2 + \frac{\omega^2}{c^2}\right) e^{i\omega t - i\vec{k}\vec{r}}\,\varphi_{\vec{k},\omega} = \frac{1}{(2\pi)^4}\int d\omega\, d^3\vec{k}\left(-\frac{1}{\varepsilon_0}\rho_{\vec{k},\omega}\right) e^{i\omega t - i\vec{k}\vec{r}}$$

$$\Rightarrow \boxed{\varphi_{\vec{k},\omega}\left(-k^2 + \frac{\omega^2}{c^2}\right) = -\frac{1}{\varepsilon_0}\rho_{\vec{k},\omega}}$$

$$\boxed{\varphi_{\vec{k},\omega} = \frac{\rho_{\vec{k},\omega}}{\varepsilon_0\left(k^2 - \frac{\omega^2}{c^2}\right)}}$$ тражимо $\rho_{\vec{k},\omega}$

$$\rho(\vec{r},t) = q\,\delta^{(3)}(\vec{r} - \vec{v}t) \quad ; \quad \rho_{\vec{k},\omega} = \int d^3\vec{r}\,dt\; q\,\delta^{(3)}(\vec{r} - \vec{v}t)\, e^{-i\omega t + i\vec{k}\vec{r}}$$

$$\rho_{\vec{k},\omega} = \int_{-\infty}^{+\infty} dt\; q\, e^{-i\omega t + i\vec{k}\vec{v}t} = q\int dt\, e^{-it(\omega - \vec{k}\vec{v})} = \boxed{q\,2\pi\,\delta(\omega - \vec{k}\cdot\vec{v})}$$

$$\boxed{\varphi_{\vec{k},\omega} = \frac{q\,2\pi\,\delta(\omega - \vec{k}\cdot\vec{v})}{\varepsilon_0\left(k^2 - \frac{\omega^2}{c^2}\right)}}$$

$$\varphi(\vec{r},t) = \frac{1}{(2\pi)^4}\int d\omega\, d^3\vec{k}\; \frac{q\cdot 2\pi\,\delta(\omega - \vec{k}\vec{v})}{\varepsilon_0\left(k^2 - \frac{\omega^2}{c^2}\right)}\, e^{i\omega t - i\vec{k}\vec{r}}$$ нека је $\vec{v} = v\vec{e}_x$!

$$= \frac{q}{(2\pi)^3\varepsilon_0}\int d^3\vec{k}\; \frac{e^{ik_x vt - ik_x\cdot x - ik_y y - ik_z\cdot z}}{k_x^2 + k_y^2 + k_z^2 - \frac{k_x^2 v^2}{c^2}}$$

$$k_x^2\left(1 - \frac{v^2}{c^2}\right) + k_y^2 + k_z^2 = \tilde{k}^2 \quad ; \quad \vec{\tilde{k}} = \left(k_x\sqrt{1 - \frac{v^2}{c^2}},\, k_y,\, k_z\right) \quad ; \quad d^3\vec{\tilde{k}} = \sqrt{1 - \frac{v^2}{c^2}}\, d^3\vec{k}$$

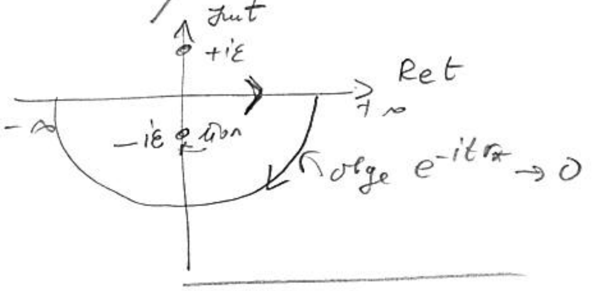
$$-ik_x(x - vt) - ik_y\cdot y - ik_z\cdot z = -i\vec{\tilde{k}}\cdot\vec{r}_* \quad ; \quad \vec{r}_* = \left(\frac{x - vt}{\sqrt{1 - v^2/c^2}},\, y,\, z\right)$$

$$\varphi(\vec{r},t) = \frac{q}{(2\pi)^3\varepsilon_0}\int d^3\vec{\tilde{k}}\,\frac{1}{\sqrt{1 - v^2/c^2}}\cdot\frac{e^{-i\vec{\tilde{k}}\cdot\vec{r}_*}}{\tilde{k}^2 \underset{\text{регулар.}}{+\varepsilon^2}} = \frac{q}{(2\pi)^3\varepsilon_0\sqrt{1 - v^2/c^2}}\int \tilde{k}^2 \sin\theta\, d\tilde{k}\, d\theta\, d\phi\; \frac{e^{-i\tilde{k}r_*\cos\theta}}{\tilde{k}^2 + \varepsilon^2}$$

$$\varphi(\bar r,t)=\frac{q\,2\pi}{(2\pi)^3\varepsilon_0\sqrt{1-v^2/c^2}}\int_0^{\infty}d\tilde k\,\frac{\tilde k^2}{\tilde k^2+\varepsilon^2}\,\frac{1}{i\tilde k r_*}\,e^{-i\tilde k r_*\cos\theta}\Big|_0^{\pi}=\frac{q}{(2\pi)^2\varepsilon_0\sqrt{1-v^2/c^2}}\,\frac{1}{ir_*}\int_0^{\infty}\frac{\tilde k}{\tilde k+\varepsilon^2}\Big(\underbrace{e^{i\tilde k r_*}}_{\tilde k=-t}-e^{-i\tilde k r_*}\Big)d\tilde k$$

$$=\frac{q}{(2\pi)^2\varepsilon_0\sqrt{1-v^2/c^2}\,ir_*}\left(\int_0^{-\infty}\frac{+t}{t^2+\varepsilon^2}\,e^{-itr_*}(+dt)-\int_0^{\infty}\frac{t}{t^2+\varepsilon^2}\,e^{-itr_*}\,dt\right)$$

$$=\frac{-q}{(2\pi)^2\varepsilon_0\sqrt{1-v^2/c^2}\,ir_*}\int_{-\infty}^{+\infty}\frac{t\,e^{-itr_*}}{t^2+\varepsilon^2}\,dt$$



$$=\frac{qi}{(2\pi)^2\varepsilon_0 r_*\sqrt{1-v^2/c^2}}(-2\pi i)\,\mathrm{Res}_{-i\varepsilon}\left(\frac{t\,e^{-itr_*}}{\underbrace{t^2+\varepsilon^2}_{(t+i\varepsilon)(t-i\varepsilon)}}\right)$$

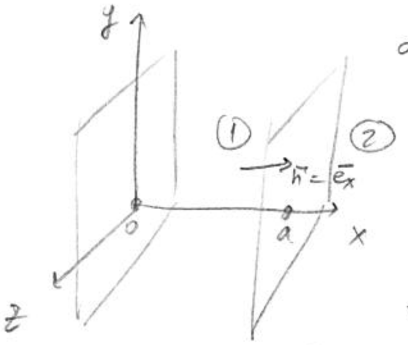
$$=+\frac{q}{2\pi\varepsilon_0 r_*\sqrt{1-v^2/c^2}}\,\frac{-i\varepsilon\,e^{-\varepsilon t r_*}}{-2i\varepsilon}=\frac{q}{4\pi\varepsilon_0 r_*\sqrt{1-v^2/c^2}}\cdot e^{-\varepsilon t r_*}\xrightarrow[\varepsilon\to 0]{}\boxed{\frac{q}{4\pi\varepsilon_0 r_*\sqrt{1-v^2/c^2}}}$$

слично за $\bar A(\bar r,t)$

Електромагнетни талас $\vec{E}(\vec{r},t) = E_0(x)\, e^{ik_y z - i\omega t}\,\vec{e}_y$ простире између две идеално проводне равни $x=0$ и $x=a$.

а) одредити дозвољене вредности $E_0(x)$ и дозвољене фреквенције

б) наћи површинску густину наелектрисања на равнима и расподелу површинских струја.



а) $\Delta \vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0$ задовољава таласну ј-ну:

$$\Delta \vec{E}(\vec{r},t) = \left(\frac{d^2E_0(x)}{dx^2}\, e^{ik_y z - i\omega t} + E_0(x)(-k_y^2)\, e^{ik_y z - i\omega t}\right)\vec{e}_y$$

$$\frac{\partial^2 \vec{E}}{\partial t^2} = -\omega^2 E_0(x)\, e^{ik_y z - i\omega t}\,\vec{e}_y$$

$$\Rightarrow \quad \frac{d^2 E_0(x)}{dx^2} - k_y^2 E_0(x) + \frac{\omega^2}{c^2}E_0(x) = 0 \qquad \frac{\omega^2}{c^2} - k_y^2 = \varkappa^2 > 0$$

$\varkappa^2 > 0$ јер је $E_0(0) = E_0(a) = 0$ два пута се анулира $\sim$ хармонијско реш.

$E_0(x) = A\sin(\varkappa x) + B\cos(\varkappa x)$ $\qquad E_0(0) = B = 0$; $E_0(a) = A\sin(\varkappa a) = 0 \Rightarrow \varkappa a = n\pi,\ n\in\mathbb{N}$

$$\Rightarrow \boxed{E_0(x) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi}{a}x\right)}$$

Вратимо израз за $\varkappa$ у $\frac{\omega^2}{c^2} - k_y^2 = \varkappa^2$; $\frac{\omega_n^2}{c^2} - k_y^2 = \left(\frac{n\pi}{a}\right)^2$; $\boxed{\omega_n = c\sqrt{k_y^2 + \left(\frac{n\pi}{a}\right)^2}}$

дозв. фреквенције

б) $\vec{n} = \vec{e}_x$ $\quad \vec{E}_2 = 0$; $\vec{E}_1 = E_0(x)\, e^{ik_y z - i\omega t}\,\vec{e}_y$ $\qquad E_{1x} = 0 \Rightarrow \boxed{\sigma = 0}$

$$\vec{e}_x = \vec{n} \times (\cancel{\vec{B}_2}^{\,0} - \vec{B}_1) = \mu_0 \vec{i}$$

$$\frac{\partial \vec{B}}{\partial t} = -\operatorname{rot}\vec{E} = -\operatorname{rot}\left(\sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi}{a}x\right) e^{ik_y z - i\omega_n t}\,\vec{e}_y\right)$$

$\operatorname{rot}\vec{A} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \partial_x & \partial_y & \partial_z \\ A_x & A_y & A_z \end{vmatrix}$

$$= -\left(-\partial_z\left(E_0(x)\, e^{ik_y z - i\omega_n t}\right)\vec{e}_x + \partial_x\left(E_0(x)\, e^{ik_y z - i\omega_n t}\right)\vec{e}_z\right)$$

$$= ik_y \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi}{a}x\right) e^{ik_y z - i\omega_n t}\,\vec{e}_x - \sum_{n=1}^{\infty} A_n\left(\frac{n\pi}{a}\right)\cos\left(\frac{n\pi}{a}x\right) e^{ik_y z - i\omega_n t}\,\vec{e}_z$$

$$\vec{B} = -\sum_n \frac{k_y}{\omega_n} A_n \sin\left(\frac{n\pi}{a}x\right) e^{ik_y z - i\omega_n t}\,\vec{e}_x - i\sum_{n=1}^{\infty} A_n \frac{1}{\omega_n}\left(\frac{n\pi}{a}\right)\cos\left(\frac{n\pi}{a}x\right) e^{ik_y z - i\omega t}\,\vec{e}_z$$

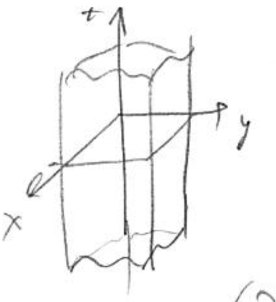
$$\vec{i} = -\frac{1}{\mu_0}\,\vec{e}_x \times \vec{B}\Big|_{x=a} = \boxed{-\frac{i}{\mu_0}\sum_{n=1}^{\infty} A_n \frac{1}{\omega_n}\left(\frac{n\pi}{a}\right)(-1)^n \cdot e^{ik_y z - i\omega t}\,\vec{e}_y}$$

Дуж правоугаоног таласовода ограниченог равнима $x=0, x=a, y=0, y=b$ простире се трансверзални магнетни талас (TM). То је талас код кога је поље магнетно нормално на правац простирања таласа (z-оса овде), па је $B_z = 0$. Овакав талас задовољава и гранични услов $E_z = 0$ на граничним равнима које су идеално проводне.

а) одредити $\vec{E}$ и $\vec{B}$ ако су они облика $\vec{E} = \vec{E}_0(x,y)\, e^{i(kz - \omega t)}$; $\vec{B} = \vec{B}_0(x,y)\, e^{i(kz-\omega t)}$

б) одредити дисперзиону релацију

в) као и површинску густину струје за произвољну моду.



$B_z=0$; E_z се анулира на гр. равнима.

$\Delta \vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}=0$ решавамо z-компоненту:

$$\Delta E_{z}(x,y)\, e^{i(kz-\omega t)} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} E_z(x,y)\, e^{i(kz-\omega t)} = 0$$

$$\left(\frac{\partial^2 E_{z}}{\partial x^2} + \frac{\partial^2 E_{z}}{\partial y^2} + \left(-k^2+\frac{\omega^2}{c^2}\right)E_{z}\right) e^{ikz-i\omega t} = 0 \qquad E_{z}(x,y) = A(x)\,B(y)$$

$$A''(x)\,B(y) + A(x)\,B''(y) + \left(\frac{\omega^2}{c^2}-k^2\right)A(x)\,B(y) = 0 \quad \Big/ \cdot \frac{1}{A(x)\,B(y)}$$

$$\underbrace{\frac{1}{A(x)}A''(x)}_{-n_x^2} + \underbrace{\frac{1}{B(y)}B''(y)}_{-n_y^2} + \frac{\omega^2}{c^2} - k^2 = 0$$

решења у осцилаторна због граничних услова

$$\boxed{k^2 = \frac{\omega^2}{c^2} - n_x^2 - n_y^2}$$

$$A(x) = C_1 \sin(n_x x) + C_2\cos(n_x x) \quad ; \quad B(y) = C_3\sin(n_y y) + C_4\cos(n_y y)$$

$E_z = 0$ за $x=0,\ x=a,\ y=0,\ y=b \Rightarrow C_2 = C_4 = 0$

$C_1 \sin(n_x a) = C_3 \sin(n_y b) = 0 \Rightarrow n_x = \frac{p\pi}{a}\ ;\ n_y = \frac{q\pi}{b},\ p,q\in \mathbb{N}$

$$\boxed{E_{z,pq} = E_{0pq}\sin\left(\frac{p\pi}{a}x\right)\sin\left(\frac{q\pi}{b}y\right)e^{ikz-i\omega t}}$$

дисп. јед. $\boxed{k^2 = \frac{\omega^2}{c^2} - \left(\frac{p\pi}{a}\right)^2 - \left(\frac{q\pi}{b}\right)^2}$

Треба наћи и E_x, E_y, B_x, B_y

$$\text{rot}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}\ ;\quad \text{rot}\,\vec{B} = \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t} \qquad \text{rot}\,\vec{A} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \partial_x & \partial_y & \partial_z \\ A_x & A_y & A_z \end{vmatrix}$$

$$\partial_y E_z - \partial_z E_y = -\frac{\partial}{\partial t}B_x \qquad \partial_y \overset{0}{B_z} - \partial_z B_y = \frac{1}{c^2}\frac{\partial E_x}{\partial t}$$

$$\partial_z E_x - \partial_x E_z = -\frac{\partial}{\partial t}B_y \qquad \partial_z B_x - \partial_x \overset{0}{B_z} = \frac{1}{c^2}\frac{\partial E_y}{\partial t}$$

$$\partial_x E_y - \partial_y E_x = -\frac{\partial}{\partial t}\underset{0}{B_z} \qquad \partial_x B_y - \partial_y B_x = \frac{1}{c^2}\frac{\partial E_z}{\partial t}$$

Заменимо:

$$\vec{E} = \vec{E}_0(x,y)\,e^{ikz-i\omega t}$$
$$\vec{B} = \vec{B}_0(x,y)\,e^{ikz-i\omega t}$$

1) $\frac{\partial E_{0z}}{\partial y} - ikE_{0y} = i\omega B_{0x}$

2) $ikE_{0x} - \frac{\partial E_{0z}}{\partial x} = i\omega B_{0y}$

3) $\frac{\partial E_{0y}}{\partial x} - \frac{\partial E_{0x}}{\partial y} = 0$

4) $-ikB_{0y} = -\frac{1}{c^2}i\omega E_{0x}$

5) $ikB_{0x} = -\frac{1}{c^2}i\omega E_{0y}$

6) $\frac{\partial B_{0y}}{\partial x} - \frac{\partial B_{0x}}{\partial y} = -\frac{1}{c^2}i\omega E_{0z}$

$$\frac{\partial E_{0z}}{\partial y} - ikE_{0y} = -\frac{i\omega^2}{c^2k}E_{0y} \Rightarrow iE_{0y}\left(k - \frac{\omega^2}{c^2k}\right) = \frac{\partial E_{0z}}{\partial y}\ ; \quad E_{0y} = \frac{-ik\,\frac{\partial E_{0z}}{\partial y}}{k^2-\frac{\omega^2}{c^2}}$$

$$\boxed{E_{y,pq} = \frac{-ik}{k^2-\frac{\omega^2}{c^2}}E_{0,pq}\left(\frac{q\pi}{b}\right)\sin\left(\frac{p\pi}{a}x\right)\cos\left(\frac{q\pi}{b}y\right)e^{ikz-i\omega t}}$$

$$\boxed{B_{x,pq} = \frac{+i\frac{\omega}{c^2}}{\underbrace{k^2-\frac{\omega^2}{c^2}}_{-\frac{p^2}{a^2}\pi^2-\frac{q^2}{b^2}\pi^2}}E_{0,pq}\left(\frac{q\pi}{b}\right)\sin\left(\frac{p\pi}{a}x\right)\cos\left(\frac{q\pi}{b}y\right)e^{ikz-i\omega t}}$$

$$ikE_{0x} - \frac{\partial E_{0z}}{\partial x} = \frac{i\omega^2}{c^2 k} E_{0x} \Rightarrow \frac{i}{k} E_{0x}\left(k^2 - \frac{\omega^2}{c^2}\right) = \frac{\partial E_{0z}}{\partial x} \Rightarrow E_{0x} = \frac{-ik \frac{\partial E_{0z}}{\partial x}}{k^2 - \frac{\omega^2}{c^2}}$$

$$\boxed{E_{x,pq} = \frac{-ik}{k^2 - \frac{\omega^2}{c^2}} E_{0,pq} \left(\frac{p\pi}{a}\right) \cos\left(\frac{p\pi}{a}x\right) \sin\left(\frac{q\pi}{b}y\right) e^{ikz - i\omega t}}$$

$$\boxed{B_{y,pq} = \frac{-i\frac{\omega}{c^2}}{k^2 - \frac{\omega^2}{c^2}} E_{0,pq} \left(\frac{p\pi}{a}\right) \cos\left(\frac{p\pi}{a}x\right) \sin\left(\frac{q\pi}{b}y\right) e^{ikz - i\omega t}} \quad \leftarrow k_{pq}$$

в) На $x=a$ $\quad \vec{n} \times (\vec{H}_2 - \vec{H}_1)\Big|_{x=a} = \vec{i}$ $\quad$ за $x=a$ $\vec{n} = \vec{e}_x$; $(\vec{H}_1)_\perp = \frac{1}{\mu_0} B_y(a)$; $\vec{B}_2 = 0$ ван таласовода

$$-\vec{e}_x \times \frac{-\frac{i\omega}{c^2}}{k^2 - \frac{\omega^2}{c^2}} E_{0,pq} \left(\frac{p\pi}{a}\right) \cos(p\pi) \sin\left(\frac{q\pi}{b}y\right) e^{ikz - i\omega t} \vec{e}_y = \mu_0 \vec{i}$$

$$\boxed{\vec{i} = \frac{\frac{i\omega}{c^2\mu_0}}{k^2 - \omega^2/c^2} E_{0,pq} \left(\frac{p\pi}{a}\right) (-1)^p \sin\left(\frac{q\pi}{b}y\right) e^{ikz - i\omega t}}$$